book problems

3.3 #20 p is equal to one of $3k$, $3k+1$ or $3k+2$. When $p=3$ we can check directly what happens, so we need only consider the case $p=3k$ for $k>1$. In this case p is not prime so we needn't check it afterall. If $p=3k+1$ then $p^2+8=3k'+9$ and hence p^2+8 is composite so we needn't consider this case either. If $p=3k+2$ then $p^2+8=9k^2+12k+4+8=3k'+12$ which is also composite. So, the only case we need to check is $p=3$ and we check to see that $3^2+8=17$, a prime, and $3^3+4=31$ which is also prime.

4.2 #13 Since $a \equiv b \pmod{n_1}$ and $a \equiv b \pmod{n_2}$ there exist e_1 and e_2 such that $a-b=e_1n_1=e_2n_2$. In particular, $a-b$ is a common multiple of n_1 and n_2. Since $a-b$ is a common multiple of n_1 and n_2 we know that the least common multiple of n_1 and n_2 divides $a-b$. (This can be seen from the proof of Theorem 2.8) or can be explicitly seen: There exist $x, y \in \mathbb{Z}$ so that $n_1x+n_2y=\gcd(n_1,n_2)$, therefore,

$$\frac{a-b}{\mathrm{LCM}(n_1,n_2)} = \frac{(a-b)\gcd(n_1,n_2)}{n_1n_2} = \frac{(a-b)xn_1}{n_1n_2} + \frac{(a-b)yn_2}{n_1n_2} \in \mathbb{Z}$$

Non-book problems:

1. $n=99991$ is prime.
2. The 100-th prime number is 541.
3. The primes less than 100000 whose last four digits are 9999 are: 49999, 59999, 79999.
4. The largest gap is 72, occurring at $g(n)=p_{n+1}-p_n$ with $n=3385$.