

book problems

- §7.3 #9 Since $\gcd(2, 77) = 1$ and $\phi(77) = 60$, by Euler's Theorem $2^{60} \equiv 1 \pmod{77}$. Therefore,
 $2^{10^5} = (2^{60})^{1666} 2^{40} \equiv 2^{40} \equiv 23 \pmod{77}$
- #11 Let p be a prime. a). Show $\tau(p!) = 2\tau((p-1)!)$. Since p is prime, $\gcd(p, (p-1)!) = 1$. Therefore,
 $\tau(p!) = \tau(p)\tau((p-1)!) = 2\tau((p-1)!)$.
Same argument for parts (b) and (c) since each of ϕ and σ are multiplicative and $\sigma(p) = p+1$ and $\phi(p) = p-1$.
- §7.4#8 For a square-free integer $n > 1$, show that $\tau(n^2) = n$ iff $n = 3$. Assume $n = 3$, then $\tau(9) = 3$ because the only divisors of 9 are 1, 3 and 9. Now assume that $\tau(n^2) = n$ for n a square free integer. Then, $n = \prod_{i=1}^r p_i$ for distinct primes p_i . Then, $\tau(n^2) = \prod_{i=1}^r \tau(p_i^2) = \prod_{i=1}^r (3)$. By assumption this equals n . Since n is square free, we must have $r = 1$ and therefore, $n = 3$.