§3.3 #10. **Solution.** $\partial f/\partial x = \sin y$ and $\partial f/\partial y = 1 + x\cos y$. The first is zero when $y = \pi k$ where k is any integer. Now $\cos(\pi k)$ equals 1 if k is even and -1 if k is odd. So we get an infininte number of critical points, namely $(x, y) = (-1, \pi k)$ if k is even and $(x, y) = (1, \pi k)$ when k is odd. Now $\partial f/\partial x = \sin y$ and $\partial f/\partial y = 1 + \cos y$ while $\partial^2 f/\partial y^2 = -x\sin y$. So the matrix of second partials is

$$\begin{pmatrix} 0 & \cos y \\ \cos y & -x\sin y \end{pmatrix}$$

The determinant is $-\cos^2 y$ so when evaluated at any of the critical points this is negative. Therefore all the critical points are saddle points.

§3.3 #22 **Solution.** A point in the plane $2x - y + 2z = 20$ looks like $(x, 2x + 2z - 20, z)$ where we substituted for y. we want to minimize $\sqrt{x^2 + y^2 + z^2}$. This is the same as minimizing $x^2 + y^2 + z^2$ which should be computationally easier to do. Substituting for y we get the function

$$g(x, z) = x^2 + (2x + 2z - 20)^2 + z^2$$

where x, z can be any real numbers. Let's find the critical points. $\partial g/\partial x = 2x + 2(2x + 2z - 20)2 = 10x + 8z - 80$ while $\partial g/\partial z = 2(2x + 2z - 20)2 + 2z = 8x + 10z - 80$. Solving

$$10x + 8z - 80 = 8x + 10z - 80 = 0$$

we get $x = 5$, $y = 5$ as the only critical point. Now the matrix of mixed partials can easily be computed to equal

$$\begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}$$

which is positive definite (i.e., $10 > 0$ and $100 - 64 > 0$). So $(5, 5)$ is a minimum for $g(x, z)$ and hence the closest point to the origin is $(x, y, z) = (5, 0, 5)$.

§3.3#34. Maximize $f(x, y) = xy$ on the rectangle $[-1, 1] \times [-1, 1]$.
Solution. We first check for critical points in the interior of the rectangle. $\nabla f = (y, x)$. This is $(0, 0)$ only if $x = y = 0$. So this is a possible extrema of the function f. But we have yet to check the boundary. We can parametrize the boundary with the four curves

$$\begin{array}{ll} c_1(t) = (t, -1) & c_2(t) = (1, t) \\ c_3(t) = (t, 1) & c_4(t) = (-1, t) \end{array}$$

where in each curve $-1 \le t \le 1$. We now look for the min and max on the boundary by checking for the min and max of $f(c_i(t))$ for each $i = 1, 2, 3, 4$. For example $f(c_1(t)) = -t$. This has minimum at $t = 1$ and max at $t = -1$. In other words the min occurs at $(1, -1)$ and the max occurs at $(-1, -1)$. Checking all four curves we see that the absolute max of f on the square occurs at $(1, 1)$ and $(-1, -1)$ and the absolute min occurs at $(1, -1)$ and $(-1, 1)$.