§2.6

#24(b). $f(c(t)) = \cos^2 t + 4\sin^2 t = 1 + 3\sin^2 t$. The max and min will occur at the critical points. In other words, when $f'(t) = 0$. $f'(t) = 6\sin t \cos$. The zeros of this function occur at $t = 0, \pi/2, \pi$. For $0 \leq t < \pi/2$, $f'(t)$ is positive and for $\pi/2 < t < \pi$, $f'(t)$ is negative. Therefore, $t = 0$ and $t = \pi$ are minimums and $t = \pi/2$ is a maximum.

$$\text{MAX:} \qquad f(\pi/2) = 4.$$

$$\text{MIN:} \qquad f(0) = 1 \qquad \text{and} \qquad f(\pi) = 1$$

25. The surface $x^2+y^2-z^2 = -1$ is the level surface of the function $f(x, y, z) = x^2+y^2-z^2$ at the value $c = -1$. Therefore, the gradient of $f(x, y, z)$ at the point $(1, 1, \sqrt{3})$ is the normal vector to the surface. $\nabla f = (2x, 2y, -2z)$ and $\nabla f(1, 1, \sqrt{3}) = (2, 2, -2\sqrt{3})$. The line through the point $(1, 1, \sqrt{3})$ in the direction $(2, 2, -2\sqrt{3})$ with velocity 10 is given by

$$L(t) = (1, 1, \sqrt{3}) + \frac{10t}{2\sqrt{5}}(2, 2, -2\sqrt{3}) = (1 + 2\sqrt{5}t, 1 + 2\sqrt{5}t, \sqrt{3}(1 - 2\sqrt{5}t)).$$

The particle hits the xy plane when $z = 0$ in other words, when $t = \sqrt{5}/10$. At this time the particle is at $(2, 2, 0)$.

§4.1

#14. $\mathbf{r}(t)$ is a path and we set $f(t) = ||\mathbf{r}(t)||$, the length of the path at time t. This is a differentiable function at all t and therefore and when a local max or local min of this function occurs we will get $f'(t) = 0$. Set $r(t) = (x(t), y(t), z(t))$. Then $f'(t) = 0$ means,

$$f'(t) = \frac{d}{dt}[\sqrt{x(t)^2 + y(t)^2 + z(t)^2}] = \frac{x(t)\frac{dx}{dt} + y(t)\frac{dy}{dt} + z(t)\frac{dz}{dt}}{||r(t)||} = \frac{r(t).r'(t)}{||r(t)||} = 0$$

So, as long as the max or min is occurring when $||r(t_0)|| \neq 0$, then $r(t_0).r'(t_0) = 0$. If the max or min occurs when $||r(t_0)|| = 0$, then $r(t_0) = (0, 0, 0)$ and we will also have $r(t_0).r'(t_0) = 0$.