§6.3

#8. Calculate the mass of the object bounded by the cylinder $x^2 + y^2 = 2x$ and the cone $x^2 + y^2 = z^2$ if the density is given by $\delta(x, y, z) = \sqrt{x^2 + y^2}$.

Call the region W. In x, y, z coordinates, this region is given by

$$0 \leq x \leq 2$$

$$-\sqrt{2x - x^2} \leq y \leq \sqrt{2x - x^2}$$
$$-\sqrt{x^2 + y^2} \leq z \leq \sqrt{x^2 + y^2}.$$

Switching to polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, this same region is given by:

$$-\pi/2 \leq \theta \leq \pi/2$$

$$0 \leq r \leq 2\cos\theta$$

$$-r \leq z \leq r.$$

Call the region in polar coordinates W^*. Note that the reason $-\pi/2 \leq \theta \leq \pi/2$ is that the equation $r = 2\cos\theta$ traces out the unit disk centered at $(1, 0)$ with period π. So we know that we want θ to range over an interval of length π. I choose $-\pi/2 \leq \theta \leq \pi/2$ so that r would always be non-negative. Using this change of coordinates we calculate:

$$\int\int\int_W \sqrt{x^2 + y^2}\, dz\, dy\, dx = \int\int\int_{W^*} r^2\, dz\, dr\, d\theta = 3\pi.$$