Exercises 6/10/08

1. Let $f(x,y) = \sum a_\alpha \mathbf{x}^\alpha$ be a convergent power series in $\mathbb{C}[[x,y]]$. Assume that f is y-general of order $m > 0$. Prove that if the Newton Polygon of f is a single point, then $f(x,y) = y^m g(x,y)$ with $g(0,0) \neq 0$.

2. With respect to which weight is $x_1^{a_1} + x_2^{a_2} + \ldots + x_n^{a_n}$ quasi-homogeneous?

3. Let $f(x,y) = \sum a_\alpha \mathbf{x}^\alpha$ be a convergent power series in $\mathbb{C}[[x,y]]$. Assume without loss of generality that f is y-general of order $m > 0$. Let $\frac{-1}{\mu_0}$ be the steepest part of the Newton polygon associated to f (assuming that the Newton polygon does not consist of just one point). Separate f into it's quasi-homogeneous parts associated to the weight μ_0. Prove that ν, the order of the lowest order terms is the intercept on the α-axis of the line through $(0,m)$ with slope $\frac{-1}{\mu_0}$ (hence $\nu = mv_0$). Why does this imply that $g(t)$ has order m in the Puiseux series expansion algorithm?

4. Compute the Puiseux expansions for
   (a) $f(x,y) = y^3 - x^5$.
   (b) $f(x,y) = y^3 - x^5 - 3x^4y - x^7$
   (c) $f(x,y) = y^4 - 2x^3y^2 + x^6$
   (d) $f(x,y) = y^4 - 2x^3y^2 - 4x^5y + x^6 - x^7$.
   (e) $f(x,y) = y^2 - x^2 - x^3$

5. Prove the following statement regarding the Puiseux Series Expansion Algorithm: If $m_1 = m$ then $\mu_0 \in \mathbb{N}$.
   Help: If you are stumped help can be found in the book Brieskorn & Knörrer.