Exercises 6/3/08 (For more on this see [CLO, chapter 3]

1. Give examples of two different elimination orders on $\mathbb{C}[x][y]$.

2. If $I = \langle x^{\alpha(1)}, \ldots, x^{\alpha(s)} \rangle$ is a monomial ideal, prove that a polynomial f is in I if and only if the remainder of f on division by $x^{\alpha(1)}, \ldots, x^{\alpha(s)}$ is zero. Knowing this, how do we solve the ideal membership problem?

3. (a) Show that $G = \{x + y, y - z\}$ is a Gröbner basis for the lex order.
   (b) Divide xy by the ordered set $(y - z, x + z)$.
   (c) Now divide xy by $(x + z, y - z)$. How can you reconcile the different quotients?

4. Show that
$$\langle x - y^{37}, x - y^{38} \rangle$$
is not a Gröbner basis with respect to lex order.

5. (a) Determine whether or not $f = xy^3 - z^2 + y^5 - z^3$ is in the ideal
$$I = \langle -x^3 + y, x^2y - z \rangle$$
.
   (b) Determine whether or not $f = x^3z - 2y^2$ is in the ideal
$$I = \langle xz - y, xy + 2z^2, y - z \rangle$$

6. (a) Find the points on the variety
$$V(x^2 + y^2 + z^2 - 1, x^2 + y^2 + z^2 - 2x, 2x - 3y - z).$$
   (b) Find the points on the variety
$$V(x^2y - z^3, 2xy - 4z - 1, z - y^2, x^3 - 4yz).$$

7. (a) Find an implicit equation for the surface pararmetrized by
$$\begin{aligned} x &= ut \\ y &= 1 - u \\ z &= u + t - ut \end{aligned}$$
   (b) Find an implicit equation for the surface parametrized by
$$\begin{aligned} x &= t + u \\ y &= t^2 + 2tu \\ z &= t^3 + 3t^2u \end{aligned}$$