

# Math 499 Exercises

## February 16, 2005

For this exercise more than ever I encourage you to work in groups. You need to do 28 computations, so it is best to split it up. We will not worry about computing Groebner Bases this week. Instead let's practice finding S-polynomials. Using lex order where  $t > x > y$ , compute the desired S-polynomials.

$$f_1 = 3t - x(1 + t^3)$$

$$f_2 = 3t^2 - y(1 + t^3)$$

$S(f_1, f_2) = ?$
-------------------

$$f_3 = \frac{1}{3}S(f_1, f_2)$$

$S(f_1, f_3) = ?$	$S(f_2, f_3) = ?$
-------------------	-------------------

$$f_4 = S(f_1, f_3)$$

$S(f_1, f_4) = ?$	$S(f_2, f_4) = ?$	$S(f_3, f_4) = ?$
-------------------	-------------------	-------------------

$$f_5 = S(f_2, f_4)$$

$$f_6 = S(f_3, f_4)$$

$S(f_1, f_5) = ?$	$S(f_2, f_5) = ?$	$S(f_3, f_5) = ?$	$S(f_4, f_5) = ?$
-------------------	-------------------	-------------------	-------------------

$S(f_1, f_6) = ?$	$S(f_2, f_6) = ?$	$S(f_3, f_6) = ?$	$S(f_4, f_6) = ?$	$S(f_5, f_6) = ?$
-------------------	-------------------	-------------------	-------------------	-------------------

$$f_7 = S(f_5, f_6)$$

$S(f_1, f_7) = ?$	$S(f_2, f_7) = ?$	$S(f_3, f_7) = ?$	$S(f_4, f_7) = ?$	$S(f_5, f_7) = ?$	$S(f_6, f_7) = ?$
-------------------	-------------------	-------------------	-------------------	-------------------	-------------------

$$f_8 = S(f_5, f_7)$$

$S(f_1, f_8) = ?$	$S(f_2, f_8) = ?$	$S(f_3, f_8) = ?$	$S(f_4, f_8) = ?$	$S(f_5, f_8) = ?$	$S(f_6, f_8) = ?$	$S(f_7, f_8) = ?$
-------------------	-------------------	-------------------	-------------------	-------------------	-------------------	-------------------

As a check:  $f_1, \dots, f_7$  should be polynomials in  $t, x, y$ 's, but  $f_8$  only has  $x$ 's and  $y$ 's.

For another example of computing these things, check out Dr. Hassett's notes:  
<http://www.math.rice.edu/hassett/CAGbook/CAGch2.pdf>