

Math 499 Exercises
February 9, 2005

1. Put the following polynomials in decreasing order with respect to lex order, graded lex order, and graded reverse lex order where $s > t > x > y$, just like the example in the notes for $f = 4x^2y^4 - 6ty + 3t^2xy - ty^5$.

(a) $6s^3x^2 - t^3 + t^2y^3 + 5s$

(b) $4t^2x + 3s^3 + s^2ty - sx^2y$

- (c) Consider the alphabet with the ordering $a > b > c > \dots > z$. We can rewrite our names as monomials as follows: Amanda = a^3dmn , Dr.Hassett = $adehrs^2t^2$. Put the members of our class in order using the three orderings.

For example Amanda $>_{lex}$ Dr.Hassett, Dr.Hassett $>_{grlex}$ Amanda, Dr.Hassett $>_{grevlex}$ Amanda.

The members are Crystal, Kristian, Lisa, Abraham, Sean, Bill, Matt, and Thomas.

2. Prove the following:

Let $f, g \in \mathbb{C}[x_1, \dots, x_n]$ be nonzero polynomials. Then:

(i) $\text{multideg}(fg) = \text{multideg}(f) + \text{multideg}(g)$.

(ii) If $f+g \neq 0$, then $\text{multideg}(f+g) \leq \max(\text{multideg}(f), \text{multideg}(g))$. Equality holds if $\text{multideg}(f) \neq \text{multideg}(g)$.

3. Consider the lemniscate $(x^2 + y^2)^2 = x^2 - y^2$. Find the parametric form of the curve, i.e. $x(t), y(t)$.