Math 499 Exercises
January 26, 2005

When plotting the following graphs on a computer, you may want let you x and y range between -4 and 4 or so.

1. Last time you found that the only singular point on the following curves is the origin, p=(0,0). Now find the multiplicity of the curves at the origin and find the tangent cones at (0,0), $TC_p(X)$. Redraw the curves you drew last week but this time also include a picture of the tangent cones.
   (This should only take you a few minutes since you already have the Taylor expansions. All you really need to do is factor the lowest terms to find the lines in the cone.) For example, for the cusp $x^3 = y^2$, you would draw a beak like figure and the line $y = 0$ doubled.

   (a) $X = C(x^4 + y^4 - x^2)$

   (b) $X = C(x^6 + y^6 - xy)$

   (c) $X = C(y^2 + x^4 + y^4 - x^3)$

   (d) $X = C(x^2y + xy^2 - x^4 - y^4)$

   (e) $X = C(x^2 + x^3 - y^2)$

   (f) $X = C((x^2 + y^2)^2 - x^2 + y^2)$

2. Use the computer to find the singular points of these new curves, the multiplicity of the singular points, and the tangent cones at the points. Draw a picture of the curves and the tangent cones at singular points. Maple will give you a really strange picture of the curves, use the tangent cones to fill in the "funny space" in the picture. Make sure to avoid singular points when using implicit plot. Otherwise your picture may not show you the singular points. A good example of where this occurs is (c).

   (a) $X = C(2x^4 - 3x^2y + y^2 - 2y^3 + y^4)$

   (b) $X = C(2y^2(x^2 + y^2) - 3y^2 - x^2 + 1)$

   (c) $X = C(2y^2(x^2 + y^2) - 2y^2(x + y) - 2y^2 - x^2 + 2x + 2y)$

   (d) $X = C((x^2 + y^2)^3 - 4x^2y^2)$