Homework #1 Due January 22.

- Read Sections 1.1 through 1.4 in Fomin & Gelfand..
- In class we discussed functions ϕ which are infinitely differentiable and have compact support, where compact means closed and bounded. These first exercises are meant to explore these ideas a little more.

1. Find a function $g \in C^\infty(\mathbf{R})$ which satisfies

$$g(x) = \begin{cases} 0 & \text{if } x \le -1 \\ 1 & \text{if } x \ge 0. \end{cases}$$

Hint: Supose that $\phi \in C^\infty(\mathbf{R})$ has $\operatorname{supp}\phi \subset [-1, 0]$ and $\phi(x) \ge 0$ for all x. What are the properties of $\psi(x) = \int_{-\infty}^{x} \phi(y)\, dy$?

2. Let $I = [a, b]$ where $a < b$. Suppose that $\epsilon > 0$. Find a function $\phi \in C^\infty(\mathbf{R})$ such that $\phi(x) = 1$ if $x \in I$, and $\operatorname{supp}\phi \subset (a - \epsilon, b + \epsilon)$.

3. Do Problem 14 on p. 32 of Fomin & Gelfand. It would be best to use ad hoc methods. Does a minimum or a maximum exist? Notice that $(y^2)' = 2yy'$.

4. Do parts a), b), and c) of Problem 15 on p. 32 of Fomin & Gelfand.