Homework #1 Due January 20.

- Read Sections 1.1 through 1.4 in Fomin & Gelfand.

1. Consider the function f defined on the unit interval $[0,1]$ by

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ x\sin(1/x) & \text{if } 0 < x \le 1. \end{cases}$$

(a) Is f continuous on $[0,1]$?

(b) Does f belong to $C^1[0.1]$?

- In class we discussed functions ϕ which are infinitely differentiable and have compact support. Compact means closed and bounded. The support of a function ϕ is the closure of the set where $\phi(x) \neq 0$, and is denoted by $\operatorname{supp}\phi$. The next two exercises are meant to explore these ideas a little more.

2. Find a function $g \in C^\infty(\mathbf{R})$ which satisfies

$$g(x) = \begin{cases} 0 & \text{if } x \le -1 \\ 1 & \text{if } x \ge 0. \end{cases}$$

Hint: Suppose that $\phi \in C^\infty(\mathbf{R})$ has $\operatorname{supp}\phi \subset [-1,0]$ and $\phi(x) \ge 0$ for all x. We constructed such a function in class. What can you say about $\psi(x) = \int_{-\infty}^{x} \phi(y)\,dy$ for $x \le -1$ and for $x \ge 0$?

3. Let $I = [a,b]$ where $a < b$. Suppose that $\epsilon > 0$. Find a function $\phi \in C^\infty(\mathbf{R})$ such that $\phi(x) = 1$ if $x \in I$, and $\operatorname{supp}\phi \subset (a-\epsilon, b+\epsilon)$. *Hint:* The function you constructed in Problem #2 might help.

4. Do Problem 14 on p. 32 of Fomin & Gelfand. It would be best to use ad hoc methods. Does a minimum or a maximum exist? Notice that $(y^2)' = 2yy'$.

5. Do parts a), b), and c) of Problem 15 on p. 32 of Fomin & Gelfand.