

Homework #9

Due March 31.

Read Sections 20, 21, 22, and 23 in Gelfand & Fomin.

1. Consider a particle which is attracted to the origin with a force that depends only on the distance from the origin.

(a) Show that the Lagrangian for this motion has the form

$$L(X, V) = \frac{1}{2}m|V|^2 - f(|X|)$$

where X is the position, V is the velocity, and f is some function.

- (b) Examine the quantities energy, (linear) momentum, and angular momentum. Which are conserved and which are not?
2. Consider the functional $\mathcal{F}(u) = \int_a^b xu'(x)^2 dx$. We showed in class that this functional is NOT invariant under translations in the independent variable x .

(a) Show that \mathcal{F} is invariant under translations in the variable u . This means with respect to transformations of the form

$$x^* = x, \quad u^* = u + \epsilon.$$

(b) What is the conserved quantity that is predicted by Noether's Theorem and part (a)?

(c) Use part (b) to find the extremals of \mathcal{F} .

(d) Show that \mathcal{F} is invariant under the transformations

$$x^* = (1 + \epsilon)x \quad u^* = u.$$

(e) What is the conserved quantity that is predicted by Noether's Theorem and part (d)?

3. In Exercise 3 of Homework #7 you found the extremals of the functional

$$\mathcal{F}(u) = \frac{1}{2} \int_a^b [u'(x) + ku(x)]^2 dx$$

(a) Show that \mathcal{F} is invariant under the transformations

$$x^* = x + \epsilon, \quad u^* = u + \epsilon\alpha e^{-kx}$$

for any constant α .

(b) What is the conserved quantity that is predicted by Noether's Theorem and part (a)?

(c) Use the result of part (b) for $\alpha = 0$ and $\alpha = 1$ to find the extremals.

4. Problem #6 on page 95 of Gelfand & Fomin.
5. Problem #7 on page 95 of Gelfand & Fomin.

Final Exam: The final exam will be made available on the last day of class, April 17, and it will be due on the last day of exams, April 29. To help me write the exam, I would appreciate it if you would submit a problem or two that you think would be appropriate for the exam. I will distribute all such problems to the class as a whole. However, I will maintain my prerogative to use all, some, or none of them.

The deadline for submissions is Tuesday, April 10.

There is no obligation to submit anything, and there will be no credit or discredit attached to any submission or lack thereof.