

Homework #8 Due April 8

- Read Sections 22, 23, 35, and 36 in Gelfand & Fomin.

1. Consider the functional $\mathcal{F}(u) = \int_a^b xu'(x)^2 dx$. We showed in class that this functional is NOT invariant under translations in the independent variable x .

(a) Show that \mathcal{F} is invariant under translations in the variable u . This means with respect to transformations of the form

$$x^* = x, \quad u^* = u + \epsilon.$$

(b) What is the conserved quantity that is predicted by Noether's Theorem and part (a)?

(c) Use part (b) to find the extremals of \mathcal{F} .

(d) Show that \mathcal{F} is invariant under the transformations

$$x^* = (1 + \epsilon)x \quad u^* = u.$$

(e) What is the conserved quantity that is predicted by Noether's Theorem and part (d)?

2. In Exercise 3 of Homework #8 you found the extremals of the functional

$$\mathcal{F}(u) = \frac{1}{2} \int_a^b [u'(x) + ku(x)]^2 dx$$

(a) Show that \mathcal{F} is invariant under the transformations

$$x^* = x + \epsilon, \quad u^* = u + \epsilon\alpha e^{-kx}$$

for any constant α .

(b) What is the conserved quantity that is predicted by Noether's Theorem and part (a)?

(c) Use the result of part (b) for $\alpha = 0$ and $\alpha = 1$ to find the extremals.

Final Exam The final exam will be made available on the last day of class, April 22, and it will be due on the last day of exams (which is different depending on whether or not you are a degree candidate). To help me write the exam, I would appreciate it if you would submit a problem or two that you think would be appropriate for the exam. I will distribute all such problems to the class as a whole. However, I will maintain my prerogative to use all, some, or none of them.

The deadline for submissions is Tuesday, April 15.

I assure you that there is no obligation on you to submit anything, and there will be no credit or discredit attached to any submission or lack thereof.