Homework #4 Due February 16.

- Read Chapter 2.
- Do Exercises 7 – 12, and 15 in Chapter 2.
- Exercises 11 and 12 show the intimate and very important connection between holomorphic functions and harmonic functions. I do not understand the first part of Exercise 11. Instead of what the book says, with the hypotheses in Exercise 11, prove that
$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} f(Re^{i\phi}) \frac{R^2 - |z|^2}{R^2 - 2R\mathrm{Re}(ze^{-i\phi}) + |z|^2}\, d\phi.$$
The hint in Exercise 11 still applies. Do Exercise 12 as well, using this new version of Exercise 11.
- Prove the following theorem stated in class.
Theorem: Let Ω be an open set and let γ be a piecewise smooth curve in Ω. Suppose that the function f is holomorphic in $\Omega \backslash \gamma$ and continuous in Ω. Prove that f is holomorphic in Ω.
Hint: Follow the proof of Theorem 5.5 to reduce the proof to showing that $\int_T f(z)\, dz = 0$ for triangles that intersect γ. Next, restrict yourself to triangles that intersect only the smooth portion of γ. Notice that you can approximate the smooth portions of γ by polygonal paths. If you get this far, the case where triangles contain a singularity of γ is easy.
- Hand in Exercises 7, 11, 12, 15 and the above theorem.