Contents

Preface

It has been almost five years since the publication of the first edition of *Ordinary Differential Equations, Using MATLAB* *. During this time period the Mathworks has released MATLAB 5, both the professional edition as well as the *Student Edition of MATLAB 5*. In addition, there have been several significant upgrades to MATLAB 5 (the current release is MATLAB 5.3), and, of course, the accompanying toolboxes have also seen changes and improvements. Thus, it was essential that *Ordinary Differential Equations, Using Matlab* adjust to reflect the changes and advances in the MATLAB software.

All of the commands in this second edition have been thoroughly tested with MATLAB, version 5.0, and the Symbolic Toolbox, version 2.0. All commands in the manual and accompanying the routines, `dfield5` and `pplane5`, will run equally well with later releases of MATLAB 5 and the Symbolic Toolbox. If you are currently using MATLAB 4, then you will either need to upgrade your software or consider using the first edition of *Ordinary Differential Equations, Using MATLAB*. If your version of the Symbolic Toolbox is a release prior to version 2.0, but you are using MATLAB 5 or a later release, you can skip the parts in this second edition that require the use of the Symbolic Toolbox without fearing a loss of continuity. The majority of the manual (about 90-95%) relies strictly on the kernel of routines provided by MATLAB 5 and the routines provided with the manual.

Although the software has seen vast change, some things still remain the same. We still feel that there is no elementary college level mathematics course for which computer graphics is more useful to the student than the ordinary differential equations course. This manual reflects our ongoing efforts to institute a significant computer component in the ODE courses taught at Rice University and College of the Redwoods.

The manual is designed to be used by the student while working at the computer.

---

* MATLAB is a registered trademark of
The MathWorks, Inc.
For MATLAB product information, please contact
The MathWorks, Inc.
24 Prime Park Way
Natick, MA 01760-1500
Phone: 508-647-7000
Fax: 508-647-7101
Email: info@mathworks.com
Web: www.mathworks.com

The student should follow the manual, executing the commands as they occur in the text. Pains have been taken to make it as easy as possible for the student to work through the manual with a minimum of assistance. Of course, some assistance will inevitably be required.

Understand that the manual is meant to be a supplementary text for students in an ODE course. It is not a textbook, nor was it written as a companion for any particular textbook. In fact, this manual can accompany most ODE textbooks (we'd like to think all), and we leave the choice of text to the instructor. Although the manual is basically a book on how to use and solve ODEs with MATLAB, there are theorems stated, and there are theoretical results quoted in the manual. However, there are no proofs.

With a plethora of excellent ODE software packages to choose from, we have often been asked why we chose MATLAB for our students. We wanted our students to spend their time learning about ODEs and not about software. That meant that the software should be as easy as possible to use. Another consideration, possibly in conflict with the first, was that if our students were going to spend time learning software, then it would be desirable if that knowledge were useful in other venues after the course was over. These thoughts led us to look closely at the mathematical computer programs MATLAB, Maple, and Mathematica.

After spending a lot of time working with these three, we decided that MATLAB was by far the easiest to use. There were two drawbacks. First, MATLAB did not, at that time, have a symbolic computing capability, but we decided that that was not as important as the ability to solve ODEs, and to graphically display the solutions. MATLAB does this better than the other two programs. Furthermore, the Symbolic Toolbox has since become available from The MathWorks, providing an interface to the symbolic computing power of Maple. The second drawback was more serious. When teaching ODEs, it is highly desirable to have easy-to-use routines that allow students to observe the direction fields of first order ODEs, and at a more advanced level, to observe the vector fields of planar autonomous systems. MATLAB had neither of these, but we had been using MATLAB for some time, and we knew that it would not be difficult to provide these routines for our students by writing programs in MATLAB.

So we chose MATLAB. We have been pleased with the results. By all indications, our students have been pleased as well.

The emphasis in the manual is on ODEs. The features of MATLAB are explained only in so far as they apply to the study of ODEs. Given that as a guiding principle, the user of this manual will nevertheless learn a great deal about MATLAB in the process. There is no programming required in the use of the manual, unless you call

```
function xpr = xsqmt(t,x)
xpr = x^2 - t;
```

a program. We do not. There are some exercises that require what we do call programming, but in these cases the code is provided.

The first chapter is extremely elementary. It is meant for the user with almost no experience with computers.

We believe that the early introduction of computer drawn direction fields and solution trajectories is an essential ingredient in a modern course on differential equations. We like to get our students acquainted with `dfield5` as early as the first day of instruction. Consequently, chapter two describes the MATLAB function `dfield5`, which displays the direction field of a single, user-provided ODE of first order. This function is very easy to use, and does not require any knowledge of how MATLAB works beyond knowing how to start it.

The third chapter is fundamental to almost everything that follows. It is here that the workings of MATLAB are explained. It is not necessary at this point to dive too deeply into the depths of MATLAB syntax. It is only necessary to understand how MATLAB works with matrices, and how to write function M-files of minimal complexity (like the one shown above). The graphical features of MATLAB are explored. Students learn this material easily.

Chapter four uses MATLAB to explore numerical methods of solving ODEs. Using MATLAB, it is quite easy to explore the errors actually made when the standard one-step solution methods, such as Euler and Runge-Kutta, are used.

In chapter five we discuss advanced use of the `dfield5` routine introduced in Chapter 3. Engineering students will find this chapter particularly useful, because the chapter highlights and explains forcing functions typically encountered in engineering courses (square waves, pulse trains, etc.).

Chapter 6 now reflects the pedagogical reality that systems of ODEs are being introduced to students much earlier in modern course syllabi. This chapter now introduces elementary use of `pplane5`, a routine designed to solve planar, autonomous systems of ODEs. We deliberately put off advanced discussion of `pplane5` until Chapter 11.

Without exaggeration, chapter seven is one of the most important chapters in the manual. It is here that MATLAB's suite of ODE solvers is described, and where the student will learn how to solve essentially arbitrary systems of ODEs numerically. Learning these skills should be a goal of a modern ODE course. Given a choice, it is almost certain that most students will choose this analysis tool in later courses and job experiences.

Chapter eight introduces students to the Symbolic Toolbox. Although this interface to the symbolic computing power of Maple can perform extraordinary tasks, we concentrate solely on its performance in the solution of ODEs and systems of ODEs.

Chapter nine discusses the linear algebra required to master the use of MATLAB to solve linear systems of equations. Many ODE books assume that students know this important topic, an assumption which is often unwarranted. For that reason the discussion in chapter nine is fairly complete.

In chapter ten we apply the material learned in chapter nine to the solution of homogeneous, linear systems of ODEs with constant coefficients. The ease with which MATLAB does matrix algebra allows students to experience a wider variety of systems than they would without the use of MATLAB. In this second edition, we now feature extensive discussion of the exponential matrix and its use in the solution of linear systems.

In chapter eleven, we return to discuss advanced use of `pplane5`, the MATLAB routine first introduced in chapter six. This chapter features discussion of nullclines and non-linear, planar, autonomous systems. Here we introduce the Jacobian matrix and linearization, separatices, basins of attraction, and a set of problems designed to help students list all possible classifications of the equilibrium point of linear, planar, autonomous systems. The trace-determinant plane is used as a vehicle for classification.

The order of the chapters is close to the order of the topics in many ODE books. However, the prerequisites for the chapters are frequently minimal. Here is a complete list of the major dependencies:

$$1 \to 2$$
$$1 \to 3 \to 8$$
$$1 \to 3 \to 4 \to 7$$
$$1 \to 2 \to 3 \to 5$$
$$1 \to 3 \to 9 \to 10$$
$$1 \to 3 \to 6$$
$$1 \to 3 \to 6 \to 9 \to 10 \to 11$$

The notation $1 \to 3 \to 8$ means cover chapter one before attempting chapter 3, and cover chapter 3 before attempting chapter 8. It is clear that the material can be covered in many orders. This having been said, it must be added that toward the end of many of the chapters there is more advanced material, especially in the exercises in chapters seven, ten, and eleven.

This second edition has seen a significant increase in the number of exercises offered at the end of each chapter. We especially wanted to provide more routine problems, so that students could gradually build their confidence before tackling harder problems and what should more properly be called computer projects. We have added problems involving ODEs with forcing functions that are commonly encountered in engineering courses, and we have included a nice block of problems whose goal is to help students classify equilibrium points of linear systems. Not all of the questions are directly computer related. Many of them require the student to use the computer to make a conjecture, and then to verify their conjecture analytically. Where a problem requires the student to use aspects of MATLAB which are not explained in the text, these are explained as part of the problem.

There is special software that is needed for the manual. This includes the functions `dfield5` and `pplane5` described in chapters two, six, and eleven, together with their associated solvers, the solver routines `eul`, `rk2`, and `rk4` used in chapter four, and the square wave function `sqw` used in chapter 5. These routines do not come with either the professional or the student version of MATLAB.

The best source for the software is the webpage http://math.rice.edu/ dfield. There you will find versions which are compatible with all versions of MATLAB going back to 3.5. This is also the location where the most up to date versions of `dfield` and `pplane` will be found in the future. In addition this is the place where you can experiment with the new java based versions.

For the best compatibility with this document, you should use the versions which are meant to be used with MATLAB ver. 5 and later.

There are many people who contributed to this project. We should start with James L. Kinsey, former Dean of the Wiess School of Natural Sciences at Rice, who originally suggested this project, and who made available resources which made quick progress possible. Ken Richardson taught the new ODE course at Rice in conjunction with author Polking for the first year, and collaborated on every aspect of it. Joel Castellanos assisted with the programming at one stage, and served as a volunteer "labie."

Over the past six years, we have had conversations with many mathematicians about this manual and the associated software. We would like to mention Henry Edwards, Herman Gollwitzer, Larry Shampine, Bob Devaney, Bob Williams, Al Taylor, Marty Golubitsky, and Beverly West. To those whom we did not mention, we offer our gratitude and apologies. Special thanks go to Larry Shampine and Mark Reichelt, who allowed us to have an early view of their MATLAB ODE suite, which is partially described in Chapter seven.

The manuscript was prepared using YandY TEX, using a modified set of macros originally written by Jim Carlson. Michael Spivak was always ready with an answer to our many questions about TEX.

The students at Rice and College of the Redwoods deserve our special gratitude. They suffered through years of experimentation, complaining only when appropriate. They were very active participants in the preparation of the manual in this form.

David Arnold — Eureka, California

John C. Polking — Houston, Texas

March 1999