

Math 211: Ordinary Differential Equations
Instructor: Qiongling Li
Final
Summer 2012

Please print name:

When you are finished with the exam, please sign the honor code below.
On my honor, I have neither given nor received any unauthorized aid on this exam:

This exam is intended to be taken in 3 hour. No books, notes or any other form of aid is permitted. There are 100 possible points. You must show all of your work, justifying when necessary. Partial credit will be given.

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(#1: 8 points) Find solution of the initial value problem
 $xy' + 2y = \sin x, y(\frac{\pi}{2}) = 0$.
Discuss the interval of existence.

(#2: 8 points) Find general solution of differential equation
 $(xy - 2)dx + (x^2 - xy)dy = 0$.
(Hint: You may use integrating factor.)

(#3: 6 points) Find general solution of differential equation
 $(y^2 - xy)dx + x^2dy = 0$.

(#4: 8 points) Find general solution of the system $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix}$. And draw the phase portrait precisely.

(#5: 8 points) Find general solution of the system $\mathbf{x}' = A\mathbf{x}$,
where $A = \begin{pmatrix} 3 & 1 \\ 3 & 5 \end{pmatrix}$. And draw the phase portrait precisely.

(#6: 10 points) Find general solution of the system $\mathbf{x}' = A\mathbf{x}$,
where $A = \begin{pmatrix} -1 & -2 & 1 \\ 0 & -4 & 3 \\ 0 & -6 & 5 \end{pmatrix}$.

(#7: 10 points) Find the solution of the initial value problem
 $y''' - 2y' + 4y = 0$, with $y(0) = 1, y'(0) = -1, y''(0) = 0$.

(#8: 8 points) Suppose that x is a solution to the initial value problem $x' = \frac{x^3 - x}{1 + t^2 x^2}$ and $x(0) = \frac{1}{2}$. Show that $0 < x(t) < 1$ for all t for which x is defined.

(#9: 10 points) Find general solution of $y'' - 2y' = 2te^{2t}$.

- (#10: 8 points) For given $A = \begin{pmatrix} -1 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 2 & -2 \end{pmatrix}$, it only has one eigenvalue λ ,
- (a) Find the smallest k such that $(A - \lambda I)^k = 0$.
 - (b) Calculate e^A .

- (#11: 10 points) For given $A = \begin{pmatrix} 3 & 10 \\ 0 & -2 \end{pmatrix}$
- (a) Calculate the eigenvalues λ_1, λ_2 and corresponding eigenvectors v_1, v_2 .
 - (b) Define the matrix $P = [v_1, v_2]$, Prove that $A = P \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} P^{-1}$.
 - (c) Calculate e^A .

(12: 6 points) Prove $\{1, \cos x, \sin x, \cos 2x, \sin 2x\}$ are linearly independent.
(Remark: In fact, $\{1, \cos x, \sin x, \cos 2x, \sin 2x, \dots, \cos nx, \sin nx, \dots\}$ are linearly independent, act as a basis for functions.)