

## PRACTIC EXAM FOR MIDTERM 2

Some remarks about Midterm 2(on Nov 13th): The total score is 100 pts. You will have 8 problems on real exam. Each part to be tested is inside homework. All contents we have taught before Midterm 2 are covered in test. On Monday(Nov 12rd), we have a review session.

The below is a "practice" version for midterm 2. There are 2 pages with 11 problems. Whenever I need you draw pictures, please draw as precisely as you can.

1. (1) Find determinnat of  $A = \begin{pmatrix} -3 & 2 & 8 \\ 1 & 4 & 1 \\ 3 & 2 & 1 \end{pmatrix}$

(2) Find  $A^{-1}$ , for  $A$ .

2. Find the basis of  $\text{Span}\{v_1, v_2, v_3, v_4\}$ ,

where  $v_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 3 \end{pmatrix}$ ,  $v_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 3 \end{pmatrix}$ ,  $v_3 = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix}$ ,  $v_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ .

3. Find a particular solution to  $y'' + 2y' + 4y = 3\cos 2t + \cos 3t$ .

4. Determine  $A = \begin{pmatrix} 4 & 2 & 8 \\ 1 & 0 & 1 \\ 3 & 5 & 1 \end{pmatrix}$  is singular or not.

5. Determine that  $y_1(t) = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix}$ ,  $y_2(t) = \begin{pmatrix} t^2 \\ \cos t \\ \sin t \end{pmatrix}$ ,  $y_3(t) = \begin{pmatrix} e^t \\ t \\ 2t \end{pmatrix}$  are linearly dependent or not.

6.(1) Solve  $y' = Ay$ , where  $A = \begin{pmatrix} -1 & -1 \\ 1 & -3 \end{pmatrix}$ ,  $y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ .

(2) Draw the solution curve portait on the phase plane.

7. (1) Solve  $y' = Ay$ , where  $A = \begin{pmatrix} -1 & -2 \\ 1 & -4 \end{pmatrix}$ ,  $y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ .

(2) Draw the solution curve portait on the phase plane.

8.(1) Solve  $y' = Ay$ , where  $A = \begin{pmatrix} 0 & 1 \\ -3 & 2 \end{pmatrix}$ ,  $y(0) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ .

(2) Draw the solution curve portrait on the phase plane.

9. Use Trace and Determinant to sketch the solution portraits of  $y' = Ay$  on the phase plane.

(1)  $A = \begin{pmatrix} 8 & 1 \\ 1 & -4 \end{pmatrix}$ . (2)  $A = \begin{pmatrix} -1 & -2 \\ 0 & -4 \end{pmatrix}$ . (3)  $A = \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix}$ .

10.(1) Draw the direction field of  $y' = Ay$ , where  $A = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$ .

(2) Sketch the solution curve to the initial value problem

$$y' = Ay, y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

11. Find general solutions to  $y'' + 4y' + 4 = te^t$ .