

PRACTIC EXAM FOR MIDTERM 1

Some remarks about Midterm I (on Sep 25th): The total score is 100 pts. You will have 8 problems. Every part to be tested is inside homework. All contents we have taught before Midterm I are covered in test. On Friday (Sep 21st) and Monday (Sep 24), we have review sessions.

I won't be at school after Friday's class (Sep 21) until the end of next week (Sep 29). But my classmate Andy will substitute me for next week's class.

The plan for next week is that: Monday is also review session and Wednesday, Friday are for new materials. Andy will have an extra office hour for you on Monday (Sep 24) afternoon 3:00-4:00pm in Hermann Brown building basement office 36.

The below is a practice version for midterm I.

1. Find solution of the initial value problem

$$(1+t)x' + x = \cos t, x(-\frac{\pi}{2}) = 0.$$

And find the interval of existence of the solution.

2. Find solution of differential equation $ydx + (x^2y - x)dy = 0$.

3. Draw direction field of $y' = ty^2$ (use both methods) and sketch the integral curves of this direction field.

4. Find general solution of $(3x + y)dx + xdy = 0$.

5. Suppose that y is a solution to the initial value problem

$$y' = (y - 3)e^{\cos(x+y)} \text{ and } y(1) = 1.$$

Show that $y(t) < 3$ for all t which x is defined.

6. Solve the linear system $Ay = b$, where

$$A = \begin{pmatrix} 8 & -6 & 9 & 8 & -1 \\ -9 & 5 & -7 & 9 & 0 \\ 1 & -4 & 1 & -3 & -7 \end{pmatrix} \text{ and } b = \begin{pmatrix} 15 \\ -30 \\ 9 \end{pmatrix}.$$

7. Show that $y(t) = 0$ and $y(t) = \frac{1}{16}t^4$ are both solutions of the initial value problem $y' = ty^{\frac{1}{2}}$, where $y(0) = 0$. Explain why this fact does not contract with the existence theorem.

8. Find the general solutions of the equation $y' = xy^2 + x$.