

### HOMWORK 4-PART C

1. Consider  $x' + 2x = \sin t$ .
  - (a) Let  $x(t)$  and  $y(t)$  be two solutions. What is the upper bound on the separation  $|x(t) - y(t)|$  predicted by the theorem (continuous dependence)?
  - (b) Find the solution  $x(t)$  with initial value  $x(0) = -\frac{1}{5}$ , and the solution  $y(t)$  with initial value  $y(0) = -\frac{3}{10}$ . Does the separation  $x(t) - y(t)$  satisfy the inequality found in part (a)?
2. Let  $x_1(t)$  and  $x_2(t)$  be solutions of  $x' = x^2 - t$  having initial conditions  $x_1(0) = 0$  and  $x_2(0) = \frac{3}{4}$ . Use Theorem (continuous dependence) to determine an upper bound for  $|x_1(t) - x_2(t)|$ , as long as the solutions  $x_1(t)$  and  $x_2(t)$  remain inside the rectangle defined by  $R = \{(t, x) : -1 < t < 1, -2 < x < 2\}$ .
3. Suppose that  $x$  is a solution to the initial value problem  $x' = x - t^2 + 2t$  and  $x(0) = 1$ . Show that  $x(t) > t^2$  for all  $t$  for which  $x$  is defined.