The goal is to understand how to shift a function to get a new one. For example, we want to know what \( f(x) = x+1 \) looks like versus \( g(x) = (x-3)+1 \) or \( h(x) = (x+3)+1 \). Notice that I’ve set this up so that \( g(x) = f(x - 3) \) and \( h(x) = f(x + 3) \).

The rule is the following: for a function \( f \) and a constant \( c > 0 \), to obtain the graph of \( f(x - c) \), we shift the graph of \( f(x) \) to the \textbf{RIGHT} by \( c \) units. On the other hand, to obtain the graph of \( f(x + c) \) we shift the graph of \( f(x) \) to the \textbf{LEFT} by \( c \) units.

Let’s think about why this rule works. We can use the above example of \( f(x) = x+1 \) and \( g(x) = (x-3)+1 = f(x-3) \) and \( h(x) = (x+3)+1 = f(x+3) \). \( f \) is a linear function, with slope \( m = 1 \) and \( y \)-intercept 1. Also, a quick calculation shows that \( x = -1 \) is a zero for the function.

\[
0 = f(x) = x + 1 \Rightarrow x = -1
\]

Because a polynomial of degree \( n \) can have no more than \( n \) real zeros, we know that \( x = -1 \) is in fact the ONLY zeros for the function. Are you with me? It might help you to draw the graph of \( y = f(x) = x + 1 \) now.

So, what I’m going to say is: we can tell how the graph is shifting by how the zero of the function shifts. SO: we calculate the \( x \)-intercepts (zeros) of our shifted functions and compare.

\[
0 = g(x) = (x - 3) + 1 = x - 2 \Rightarrow x = 2
\]

\[
0 = h(x) = (x + 3) + 1 \Rightarrow x = -4
\]

So the line \( g(x) \) crosses the \( x \)-axis at \( x = 2 \) which is \textbf{three units to the right of the zero for} \( f(x) \), so the whole line is shifted three units to the right. The line \( h(x) \) crosses the \( x \)-axis at \( x = -4 \) which is \textbf{three units to the left of the zero for} \( f(x) \), and thus the whole line \( h(x) \) is shifted three units to the left of \( f(x) \). And this lines up with the rule above. Yay!