We talked about \textbf{limits} today in class. I wanted to make a couple of points:

- When we talk about the limits, we are always talking about the limit of a function \textbf{at a specific point...and the point that we are looking at matters!} Notice: \(\lim_{x \to 3} \frac{1}{(x-3)^2}\) does not exist (because the right and left hand limits \(\lim_{x \to 3^-} \frac{1}{(x-3)^2}\) and \(\lim_{x \to 3^+} \frac{1}{(x-3)^2}\) do not agree). However, one can show that \(\lim_{x \to 4} \frac{1}{(x-3)^2} = 1\) (we’ll learn why this is true on Wednesday).

- I know that I said this many times in class, but I thought that giving you a algorithmic procedure for checking whether or not a limit exists would be helpful: Let \(f\) be a function, and suppose we want to know: does \(\lim_{x \to a} f(x)\) exist? and if so, what is its value? I recommend drawing the graphs of some slightly modified fundamental functions, and following the procedure below:

1. Pick the value \(L\) that you think the value of the limit as \(x\) approaches \(a\) should be (like we did in class on Friday - pick something reasonable).
2. Next, pick some (probably teensy-tiny) real number \(h\). For example: \(h = \frac{1}{100}\).
3. Now consider the values \([L-h, L+h]\). This is a horizontal strip across the \(xy\)-plane, and the possible value of our limit is precisely in the middle of the strip. Zoom in around \(x=a\), and answer the following question: can we find another small number \(d\) such that if you pick any \(x\)-value in \([a-d, a+d]\), then value \(f(x)\) will land in that horizontal strip \([L-h, L+h]\)?
4. You either answered yes or no to the above question.

If you answered \textbf{no} to the question above, then there are two possibilities:

(a) You’ve picked the wrong value for \(L\). In other words, the limit might exist, but it isn’t equal to the value that you’ve picked. So: restart the algorithm, picking a new value for \(L\) in step 1. If it works, great! If not...see the next possibility

(b) The limit of \(f(x)\) as \(x\) approaches \(a\) does not exist. (Think about why this happens if \(f\) is a piecewise defined function with a big jump at \(x = a\), or if \(f(x) = \sin(\pi/x)\).) In particular, if for every value of \(L\) that you pick, you answer \textbf{no} to
the question in step 3, then the limit of $f(x)$ as $x$ approaches $a$ does not exist.

On the other hand, if you answered yes to the question above, then you have to go back to question 2, and make sure that you can answer yes for any value $h$ that you pick. If you can always answer yes...then $\lim_{x\to a} f(x) = L$ and you’re done!