This assignment is designed to test your understanding of the concepts underlying many of the computations in Chapter 3. In a computational problem, be sure to justify your steps by citing a theorem or writing an explanatory sentence if necessary. This is NOT an optional assignment, it is due Monday, March 15, 2010 at the beginning of class. Recall that I do not accept late assignments. Please see your syllabus for information on how written assignments are graded. A bonus assignment will be posted soon. You should turn in the bonus assignment on paper that is separate from this assignment!

In your own words, complete all 5 of the following problems:

1. Prove the following:
   (a) If \( f(x) = \sec^{-1}(x) \), then \( f'(x) = \frac{1}{x\sqrt{x^2-1}} \).
   (b) If \( j(x) = \tanh(x) \), then \( j'(x) = \text{sech}^2(x) \)

2. The number \( e \) in action!
   (a) Give two definitions of the number \( e \). Explain why natural logs are used more frequently in calculus than \( \log_a \) where \( a \neq e \).
   (b) Solve:
      i. If \( g(x) = a^x \) for an arbitrary constant \( a \), then \( [g(x)]^n =? \)
      ii. Does there exist a value \( a \) for which \( [g(x)]^n = 0 \) for all \( n \)? If so, what is it? If not, why not?
      iii. If \( g(x) = a^x \) for an arbitrary constant \( a \), then \( g^{(n)}(x) =? \)
      iv. Does there exist a value \( a \) for which \( g^{(n)}(x) = 0 \) for all \( n \)? If so, what is it? If not, why not?
   (c) What is a differential equation? Explain how to obtain a solution to the law of natural growth.
   (d) Suppose that you want to pay cash for a watch (whose price never changes). You earn a bonus at work equal to half the price of the watch. If you invest your bonus with interest rate 6 percent compounded continuously, how long is it until you can buy the watch? What is the equivalent annual interest rate the bank would need to offer you to buy the watch at the same time as if you invested it with the compounded interest rate above?

3. A lighthouse is located on a small island 3 km away from the nearest point \( P \) on a straight shoreline and its light makes 4 revolutions per
minute. How fast is the beam of light moving along the shoreline when it is 1 km from $P$? Be sure to include a clearly labeled diagram, the known/unknown quantities, and their relationship in your solution.

4. Answer the following in your own words.

(a) Give a specific example (i.e., write out a formula) of a function $f$ and a point $x = a$ for which linear approximation does a poor job of approximating the value $f(a)$ of the function.

(b) Draw a diagram that illustrates the relationship between a function $f$, its derivative $f'$, its differential $dy$ and the changes $\Delta x$ and $\Delta y$.

5. Let $y = (\ln(x))^{\cos(x)}$. Use logarithmic differentiation to find $dy/dx$. 