THINGS TO DO WITH A GEOBOARD

The following list of suggestions is indicative of exercises and examples that can be worked on the geoboard. Simpler, as well as, more difficult suggestions can easily be tested. The list given here is not exhaustive. Insofar as possible items included in this list were selected to encourage extension of the limits of the students’ imagination, beguiling with suggestions while hoping for discovery of valuable principle.

The geoboard has a severe limitation in that curvilinear constructions are not feasible on this device. A special geoboard can be put together for curvilinear constructions.

This particular list was built around a geoboard using 25 nails arranged in one-inch squares to obtain 16 square units. Each square unit is marked off in one-half inch divisions to facilitate estimates of distances.

This is intuitive geometry. Any figure that can be constructed by rubber bands placed around the nails is fair game. Equal lengths, equal angles, parallel lines, perpendicular lines, etc., help build spatial imagery and geometric concepts. From these beginnings, more difficult and provocative problems should develop.

1. Construct a square, 4 units on a side.
2. Construct a square, 4 square units.
3. Construct a triangle, 4 square units.
4. Construct a rectangle, 3 square units.
5. Construct a rectangle, 2 units on one side, 3 units on the other side.
   a. What is the area of this rectangle?
   b. What is the perimeter of this rectangle?
   c. Divide the area into 2 parts — are the areas of the parts equal? If they are not equal, which area is larger? How can you express this: one area > other area, one area < other area, one area = X other area, one area – other area = ?
   d. Divide the area into equal thirds by means of another colored rubber band and then divide that area into equal halves. Can you find 1/6 of the total area? 2/3? 5/6? Can you show on the geoboard that 1/2 X 1/3 = 1/6?
6. Repeat No. 5, now by constructing a rectangle whose area equals 12 square units. What other mathematical relations can you discover? Can you show on the geoboard that 5/6 – 1/3 =1/2? 1/2 – 1/3 = 1/6? Anything else?
7. Construct a triangle of area equal to 3 square units.
8. Construct a parallelogram of area equal to 3 square units.
9. Construct a trapezoid of area equal to 5 square units.

10. Construct a pentagon. What is its area?

11. Construct a hexagon of area equal to 4 square units. Can you construct a hexagon of twice this area?

12. Construct triangles of area equal to 1 square unit: 2, 3, 4, 5, 6, 7, 8. Are any of these impossible?

13. Construct an octagon. What is its area? Can you construct a square inside the octagon? What is its area?

14. Construct a triangle and a rectangle so that they both have equal area. Which has the smaller perimeter?

15. Construct a rectangle and a square so that they both have equal area. Which has the smaller perimeter?

16. How many different figures can you construct each equal in area? Which of these has the smallest perimeter? Which has the greatest perimeter?

17. Take a 12-inch piece of string. Try to enclose the greatest amount of area with the string by passing the string around the nails. Draw the figure on graph paper that appears to have the greatest area. What do you think the figure would be if you didn’t have so many corners to enclose? This is the famous Isoperimetric problem.

18. Two plane figures are similar if their sides are proportional lengths and their angles are equal. A square 2 inches on each side is similar to a square one inch on a side. How many similar squares can you construct on the geoboard?

19. How many similar rectangles can you construct? Is a parallelogram similar to a rectangle? Is a trapezoid similar to a parallelogram?

20. How many similar triangles can you construct? What can you observe about the angles between the sides of similar triangles?

21. Two plane figures are congruent if the sides and angles of one figure are equal to the sides and angles of the second figure. Construct two congruent squares. Are their areas equal?

22. Construct three congruent triangles.

23. Can you construct a triangle whose area is equal to that of a square? Are the square and triangle congruent? Are they similar?

24. Can you construct a triangle whose perimeter is equal to that of a square? How close can you come?
25. Construct various triangles. Can you observe any relationship between the lengths of the sides? Can you construct a triangle in which one side will be as long as the sum of the other two sides?

26. Can you construct a square whose area equals 2 square units?

27. What other geometric concepts can you develop on the geoboard?

28. Construct a right triangle equal to the area of 1/2 square unit. Construct a square on each side of the triangle. What is the area of each square? Can you observe any relationship between these squares? This is a special case of the Pythagorean Theorem. With a larger geoboard, you can construct more proofs of this famous theorem.

29. Another proof of the Pythagorean Theorem — construct 4 right triangles on the geoboard with base length equal to three and altitude equal to one. Start at the right hand corner, so your 1st triangle extends up one unit and to the left 3 units. The vertex of your next triangle extends upward 3 units and left 1 unit. The next one extends left 3 units and down 1 unit. The 4th one extends down 3 units and right 1 unit. Note that a square is formed in the corner. This is the square of the hypotenuse. Each side of the outside square equals the sum of the two sides, and so the total area formed of the outermost lengths equals the area of the inside square plus the area of the 4 triangles. Can you complete the proof of the Pythagorean Theorem?

30. Can you construct a length on the geoboard equal to: 
\[ \sqrt{2} \] \[ \sqrt{3} \] \[ \sqrt{5} \] 

NOTE: Other constructions and proofs will be discovered as you gain practice with the geoboard. Suggested exercises may stimulate your discovery of additional uses.

31. Assume each nail is represented by a pair of numbers (Cartesian Coordinates). Join the points (0, 0) and (4, 4). What is the number pair that identifies the midpoint of this line?

32. Join (0, 1) and (4, 3). What is the number pair that identifies the midpoint of this line?

33. Join (1, 3) and (3, 1). What is the number pair that identifies the midpoint of this line? Can you observe any pattern in the determination of the midpoints? How would you write such a condition in terms of a □ and a △?

34. What is the slope of the line segment in No. 31? 32? 33? 34?

35. Join (0, 0) to (4, 4). Intersect this join by a rubber band connecting (1, 3) to (3, 1). Do these line segments appear to be perpendicular? What is the slope of the second line segment? Can you discover any relationship between these slopes?

36. Keep the rubber band in place joining (0, 0) to (4, 4). Use another rubber band to join (1, 3) to (2, 0). What is the slope of this latter line segment? Does it have the same relationship discovered in No. 35? Is it perpendicular to the join of (0, 0) and (4, 4)?
37. Repeat No. 36, but instead of joining (1, 3) to (2, 0), now join (1, 0) to (4, 3). What is the slope of this latter line segment? How is it related to the slope of the line segment joining (0, 0) to (4, 4)? Are these lines perpendicular? If not, how would you describe them? What discovery have you made?

38. Use a rubber band to plot the open sentence (-1 X + 4 = Δ). Does this open sentence connect (0, 4) and (4, 0)? What is the midpoint of this line segment? Use another rubber band to connect this midpoint to (0, 0). What is the slope of this latter segment? What is the line segment from (0, 4) to (4, 0)? Are these two line segments perpendicular, parallel, or neither? What is the length of the line segment from (0, 0) to the midpoint of the line joining (0, 4) to (4, 0)? Did you use the Pythagorean Theorem to find this distance? Do you have any new discovery to announce?

39. With a rubber band, divide the geoboard points to form a right triangle of area equal to 8 square units. Note that starting from any vertex you can observe the following sequence of nails: 1, 2, 3, 4, 5 (if your geoboard has more than 25 nails you may observe more). Use smaller rubber bands to form a triangle that encloses the 1 and 2; 1, 2, and 3; 1, 2, 3, and 4; 1, 2, 3, 4, and 5. How many nails (total) are in each triangle? Is there any pattern to each of the triangle sums? If each of these triangular sums is called a triangular number, can you show that the 5th triangular number is 5 (5+1)/2? Can you discover a law?

40. Using rubber bands, form a set of squares beginning at the lower left corner of the geoboard, of area equal to 1, 4, 9, 16 square units. How many squares did you add to the area enclosed by the first rubber band to get to the next one? Can you discover a law about the square of a number and sums of odd numbers?

41. Construct a right triangle of area to 6 square units. Construct an equilateral triangle of area equal to 6 square units. Which triangle has the shorter perimeter? Try this with other areas and compare the perimeters of the equilateral triangle, the right triangle, and the isosceles triangle. Which triangle seems to have the least perimeter?

42. Surround all 25 nails with a rubber band. Let this be a picture of the set of points A. Surround any set of points interior to set A with another rubber band. Since all of the second set of points are interior to A, your representation shows A∩B.

43. Construct an illustration of A∩B; A ∪ B; A∩B∩C.

44. Construct an illustration of A ∪ B′; A ∪ B∩C; A′ ∪ B∩C.

45. A convex plane figure is one in which every pair of distinct points can be joined by a straight line segment that lies within the boundary of the figure. A concave plane figure may contain at least one pair of points, which will be joined by a straight line segment which will lie outside the boundaries of the figure. Construct examples of convex and concave plane figures.

46. An area contained by a boundary that does not cross itself is a simply closed figure. If the boundary of the area crosses itself in forming the total figure, a non-simply closed figure is formed. Construct examples of each of these.
47. A set of points, S, is said to be symmetric in a line if the set S is the same as the set of Images of the points of S; or more briefly if it is its own image. Since we think of geometric figures as sets of points, the above definition applies to figures as well. In what line or lines are the following figures symmetric? An isosceles triangle; equilateral triangle; rectangle; square?

48. If one set of three non-collinear points is the mirror image of another set in a line, then the orientation of the two sets is different, i.e., one set has clockwise orientation and the other counter-clockwise orientation. An even number of reflection does not change the orientation of a set of three non-collinear points. Can you show that a succession of an odd number of reflections is an orientation changing transformation, and a succession of an even number of reflections is orientation preserving?

This does not exhaust the possible list of exercises and examples to be performed on the geoboard. In gaining experience with this device, other ideas will no doubt arise and should be attempted. Creating new opportunities for discovery is an exhilarating by-product of the investigation suggested above. Projective geometry topology and set theory can become interesting extensions of this brief introduction. Problem No. 17 is a crude introduction to Dido’s problem and opens up the geometry of isoperimetric problems. Taken with problems 47 & 48, the Steiner reflection theorems offer intuitive analyses of isoperimetric problems, including the theorem “of all plane figures the circle has the highest I.Q.” There is also the beautifully simple “proof” that “for any plane figure which is not a circle there is another with greater area and with the same perimeter”.