

**STRENGTHENING MATHEMATICS TEACHERS' PEDAGOGICAL
CONTENT KNOWLEDGE THROUGH COLLABORATIVE
INVESTIGATIONS IN COMBINATORICS**

Ann R. McCoy
Decision Information
Resources, Inc.

Antwanette N. Hill
Decision Information
Resources, Inc.

Jacqueline J. Sack
Rice University

Anne Papakonstantinou
Rice University

Richard Parr
Rice University

The NSF-funded Rice University Mathematics Leadership Institute immersed high school lead teachers in collaborative, combinatorics problem-solving experiences during an intensive four-week summer institute. The program challenged participants' pedagogical content knowledge and their views about collaborative problem solving as evidenced by statistically significant gains in test scores, their self-reported ratings on content and problem-solving abilities, and excerpts from journal writings.

According to Freudenthal (1991), only a small minority learns when mathematics is taught as a ready-made subject in which students are given definitions, rules and algorithms from which they are expected to proceed. Instead, learners should be given opportunities to reinvent mathematics in the manner that mathematicians create mathematics. Within professional development settings, in order for teachers to reinvent mathematics in this way and experience disequilibrium as their students do, the content should be in a domain which teachers typically do not know deeply.

Through a combinatorics domain problem-solving approach, the NSF-funded Rice University Mathematics Leadership Institute (MLI) (1) provided teachers opportunities to reinvent their mathematics pedagogy and knowledge. This allowed teachers to experience firsthand the instructional approaches they were expected to use with their own students (NSDC, 2002). We used challenging mathematics content with deliberately modeled learner-centered pedagogical approaches to develop teachers' pedagogical content knowledge (Shulman, 1986).

Program Methodology and Analysis

Thirty-two high school mathematics lead teachers, participating in their second of two intensive one-month summer institutes, were immersed in mathematical experiences for which they earned graduate credit. This experience served as the catalyst for reinventing teachers' views on the learning of mathematics. We adapted problem sets in the combinatorics domain from Park City Mathematics Institute materials (Kerins, Sinwell & Matsuura, 2004). We introduced teachers to this experience using the Simplex Lock problem, which they were invited to solve by the end of the program—once they had acquired a deeper understanding of the concepts through their experiences with collaborative problem solving. Each day, teachers solved problem sets consisting of Essential Problems (for everyone to solve), Neat Problems (similar to the essential problems but reaching toward the next topic) and Challenging Problems (for those who were ready for an intellectual stretch). Teachers were not expected to complete all

problems and could solve them in any order. On the last day of the program, some teachers shared their solutions to the Simplex Lock problem. Those who had not solved it followed various solutions competently because they had acquired sufficient conceptual knowledge during the program. Table 1 shows representative problems from the first and third weeks of the program.

Table 1

Sample Problems from the 2006 MLI Summer Leadership Institute Combinatorics Strand

Essential Problems	Neat Problems	Challenging Problems
Week 1		
<p>A “train of length 5” is a row of rods whose combined length is 5.</p> <p>1. How many trains of length 4 are there?</p> <p>3. Find a formula for the number of trains of length n. Come up with a convincing reason that your rule is correct.</p>	<p>7. If there are three flavors of ice cream, how many different three-scoop cones can you make using each flavor exactly once?</p> <p>12. If you can make 220 different three-scoop bowls of ice cream, how many different three-scoop cones can you make?</p>	<p>17. What’s the mean length of car used when you make all the trains of length 5? Is there a general rule at work here? Can you justify it?</p> <p>18. How many three-scoop bowls could you make at Ben & Jerry’s if you were allowed to duplicate flavors? Is there a general rule?</p>
Week 3		
<p>1. $\{\{C\}, \{V\}\}$ means a bowl with one scoop of chocolate and a bowl with one scoop of vanilla. Here are some possible combinations: $\{\cdot\} \cdot \{\{C, V\}\} \cdot \{\{C, V, S\}\} \cdot \{\{S, V\}, \{C\}\} \cdot \{\{S\}\} \cdot \{\{V\}, \{S\}\}$. How many different desserts are possible?</p> <p>3. Without a calculator, expand $(h + t)^5$.</p> <p>5. Spend at least 15 minutes thinking about the lock problem.</p>	<p>9. In a box of 12 batteries, it is known that 5 are dead. Four batteries are selected at random. Find the probability that</p> <p>(a) exactly one dead battery is selected.</p> <p>(b) all four of the selected batteries are dead.</p> <p>(c) at most two of the selected batteries are dead.</p>	<p>12. In row 7 of Pascal’s Triangle, the numbers 7, 21, and 35 appear consecutively. Interestingly, these three numbers form an arithmetic sequence. Does this ever happen again? If so, find the next three times it happens. If not, prove it can’t happen again.</p>

We administered pre- and post-tests based on items similar to the Essential Problems. Test scores of teachers were used to conduct a paired samples t-test to measure the change in their combinatorics content knowledge as a result of participating in the summer institute. The highest score possible was 40. Teachers’ mean score on the pre-test was 10.4 (median = 11). The post-test mean score on the same measure was 33.9 (median = 36.5). The change in score was statistically significant, $t(df = 31) = -20.79, p < .0001$ (2) and suggests that teachers’ combinatorics content knowledge increased significantly as a result of participating in the summer institute.

Individuals can be transformed by critically reflecting on situations that are not consistent with their preconceived notions. Such situations or dilemmas often prompt new interpretations of experiences. Facilitating such transformative learning experiences are most effective when instructors assist learners by encouraging them to examine or question assumptions that underlie their beliefs, feelings, and actions; assess the consequences of their assumptions; identify and explore alternative assumptions; and test the validity of their assumptions through reflective dialogue (Mezirow, 2000). Providing opportunities for teachers to reflect on experiences that do

not fit their preconceived ideas about or understanding of mathematics and how it is taught is a major component of the MLI's approach to teacher development.

Collaborative settings in which participants can engage in discourse and reflection to consolidate their knowledge assists learners adapt prior knowledge to new challenges (Freudenthal, 1991) particularly in. During the MLI's combinatorics instruction, answers or solutions to problems were not provided. Instructional staff deliberately refrained from affirming answers but prompted teachers to discuss their results with their peers. Teachers' journal entries revealed their early frustrations. For example,

- *What I hate is that I have people monitoring me and cannot answer if my solution is correct.*

Later, their acceptance and excitement about the learning experience emerged:

- *The combinatorics problems are finally starting to make sense. From day one, as a group we generated answers and answers only. I had an answer but could not explain my thought process. They are more challenging every day and it challenges my understanding to not just get an answer but explain my process and thought.*
- *I worked with a new group today. It is wonderful to see how different people work on a problem.*
- *I wish I could get across to my students the satisfaction of the challenge of an advanced math problem. I want them to enjoy the struggle the way I am.*

At the conclusion of the 2006 summer institute, based on a post-program survey, over ninety percent of teachers agreed that they had increased their knowledge and understanding of how to solve combinatorics problems as well as their ability to solve mathematics problems.

Endnotes

1. This research was supported by the National Science Foundation (NSF) under grant 0412072. The views expressed do not necessarily reflect official positions of the NSF.

2. Preprogram N = 31; Postprogram N = 32—the larger postprogram N was due to the delayed arrival of one teacher during the summer institute.

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