

Unit 4 – Informal Logic/Deductive Reasoning

Informal Language

Overview: Participants learn/review some of the language and notation used in informal logic.

Objective: **TE_xES Mathematics Competencies**
III.012.A. The beginning teacher understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.
V.019.F. The beginning teacher uses appropriate mathematical terminology to express mathematical ideas.

Geometry TEKS

b.1.A. The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems.

b.3.A. The student determines if the converse of a conditional statement is true or false.

Background: Presenter will provide background information for participants.

Materials: easel paper, colored markers

New terms: argument, biconditional statement, conditional statement, contrapositive statement, converse statement, inverse statement

Procedures:

This activity introduces terms and mathematical notation that will be used later in this unit. Before participants begin work on the activity, define and apply terms with the example:

A logical *argument* consists of a set of premises and a conclusion. Example: “Mr. French is the only calculus teacher. Mr. French, Ms. Anderson, Ms. Allen, and Ms. Short teach pre-calculus.”

When you write “If I take calculus, then Mr. French is my teacher” you are writing a *conditional* statement.

“I take calculus” is the premise and “Mr. French is my teacher” is the conclusion. To create the *converse* of a conditional statement, the two parts of the conditional statement are simply interchanged.

“If Mr. French is my teacher, then I take calculus.”

Is the converse of a conditional statement always true when the conditional statement is true?

Not always, because, in our example, I could take pre-calculus from Mr. French.

The negation of a sentence is made by placing the word not in the sentence appropriately. To create the *inverse*, the two parts of a conditional are negated.

“If I do not take calculus, then Mr. French is not my teacher.”

Is the inverse of a conditional statement always true when the conditional statement is true?

Not always, because, in our example, Mr. French could be my pre-calculus teacher.

To create the *contrapositive*, the two parts of the conditional are reversed and negated.

“If Mr. French is not my teacher, then I don’t take calculus.”

Is the contrapositive of a conditional statement always true when the conditional statement is true?

Yes, it is true.

Remind participants to add the terms argument, conditional statement, converse statement, inverse statement, and contrapositive statement to their glossaries.

Have each group work on the activity page. Provide each group with easel paper and markers. After groups complete the activity page, have each group present a different problem to the entire group.

Write the given sentences (1–4) as conditional statements. Then find their converses, inverses, and contrapositives. Assuming the conditional statements are true, determine whether each of the converse, inverse, and contrapositive statements is true or false. Give an explanation for each false statement.

1. I use an umbrella when it rains.

Conditional: If it rains, then I use an umbrella.

Converse: If I use an umbrella, then it rains.

Inverse: If it does not rain, then I do not use an umbrella.

Contrapositive: If I do not use an umbrella, then it does not rain.

The converse is not always true, since I may also use my umbrella on a very sunny day.

The inverse is not always true, because if it does not rain, it may be a very sunny day and I may use my umbrella.

The contrapositive is true.

2. A rhombus is a quadrilateral with four congruent sides.

Conditional: If a quadrilateral is a rhombus, then it has four congruent sides.

Converse: If a quadrilateral has four congruent sides, then it is a rhombus.

Inverse: If a quadrilateral is not a rhombus, then it does not have four congruent sides.

Contrapositive: If a quadrilateral does not have four congruent sides, then it is not a rhombus.

The conditional is true. The converse, the inverse, and the contrapositive are true.

3. The sum of the measures of the interior angles of a triangle is 180° .

Conditional: If a polygon is a triangle, then the sum of the measures of the interior angles is 180° .

Converse: If the sum of the measures of the interior angles is 180° , then the polygon is a triangle.

Inverse: If a polygon is not a triangle, then the sum of the measures of the interior angles is not 180° .

Contrapositive: If the sum of the measures of the interior angles is not 180° , then the polygon is not a triangle.

The conditional is true. The converse, the inverse, and the contrapositive are true.

4. Vertical angles are congruent.

Conditional: If two angles are vertical angles, then they are congruent.

Converse: If two angles are congruent, then they are vertical angles.

Inverse: If two angles are not vertical angles, then they are not congruent.

Contrapositive: If two angles are not congruent, then they are not vertical angles.

The conditional is true. The converse is not always true. If an angle is bisected, then the two smaller angles are congruent but not vertical angles.

The inverse is not always true. If two right angles are adjacent, then they are not vertical, but they are congruent.

The contrapositive is true.

5. Write a real-world example of a conditional statement with a true converse.

Possible Answer:

My cat and dog always eat together.

Conditional: If my cat eats, then my dog eats.

Converse: If my dog eats, then my cat eats.

6. Write a real-world example of a conditional statement with a false converse.

Possible Answer:

Every Mathlete at Lanier Middle School is an 8th-grade student.

Conditional: If a Lanier Middle School student is a Mathlete, then he/she is an 8th-grade student.

Converse: If a Lanier Middle School student is an 8th-grade student, then he/she is a Mathlete.

We do not know that every 8th-grade student is a Mathlete.

7. What conclusions can be made about the truth of the converse, inverse, and contrapositive statements for a given conditional that is true?

When two statements are either both true or both false they form a biconditional

statement. A conditional and its contrapositive form a biconditional statement. The converse and inverse statements of a conditional statement also form a biconditional statement.

Remind participants to add the term biconditional statement to their glossaries.

Close the activity with a discussion of the van Hiele levels for this activity. Success in this activity indicates that participants are working at the Relational Level or approaching the Deductive Level, because they informally recognize relationships among a conditional statement and its contrapositive, converse, and inverse statements.

Informal Language

Write the given sentences (1-4) as conditional statements then find their converses, inverses, and contrapositives. Assuming the conditional statements are true, determine whether each of the converse, inverse, and contrapositive statements is true or false. Give an explanation for each false statement.

1. I use an umbrella when it rains.

2. A rhombus is a quadrilateral with four congruent sides.

3. The sum of the measures of the interior angles of a triangle is 180° .

4. Vertical angles are congruent.

5. Write a real-world example of a conditional statement with a true converse.

Inductive Triangle Congruence

Overview: This activity develops the triangle congruence theorems using an inductive approach.

Objective: **TE_xES Mathematics Competencies**

III.012.A. The beginning teacher understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).

III.012.D. The beginning teacher uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.

III.012.E. The beginning teacher describes and justifies geometric constructions made using compass and straight edge, reflection devices, and other appropriate technologies.

III.013.C. The beginning teacher uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).

V.018.A. The beginning teacher understands the nature of proof, including indirect proof, in mathematics.

V.018.B. The beginning teacher applies correct mathematical reasoning to derive valid conclusions from a set of premises.

V.018.C. The beginning teacher uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.

V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS

b.1.A. The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems.

b.2.B. The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

b.3.B. The student constructs and justifies statements about geometric figures and their properties.

b.3.D. The student uses inductive reasoning to formulate a conjecture.

b.3.E. The student uses deductive reasoning to prove a statement.

e.2.B. Based on explorations and using concrete models, the student formulates and tests conjectures about the properties and attributes of polygons and their component parts.

e.3.B. The student justifies and applies triangle congruence relationships.

Background: Participants should be familiar with congruent triangle properties.

Materials: unlined 8.5 in. by 11 in. paper, compass, centimeter ruler, protractor, spaghetti, scissors

New terms: deductive reasoning, inductive reasoning

Procedures:

Begin by explaining the differences between inductive and deductive reasoning. *Inductive reasoning* is the process of observing data, recognizing patterns, and making generalizations from your observations. Much of geometry uses inductive reasoning especially at the lower van Hiele levels. Discovering that the sum of the measures of the interior angles of a triangle is 180° by tearing the angles of different triangles and observing that the sum of their measures is 180° for each of the triangles is an example of inductive reasoning. Inductive reasoning is used in geometry to discovery properties of figures.

Deductive reasoning is the process of proving or demonstrating that if certain statements are accepted as true, then other statements can be shown to follow. When a lawyer uses evidence (premises) to prove his/her case (conclusion), he/she is using deductive reasoning. Deductive reasoning is used in geometry to draw conclusions from given information. Deductive reasoning is generally used at the higher van Hiele levels.

Remind participants to add the terms inductive reasoning and deductive reasoning to their glossaries.

We draw conclusions about the congruence relationship of two triangles using both types of reasoning. This activity uses an inductive approach (with examples taken from *Discovering Geometry: An Investigative Approach*, 3rd Edition, © 2003, pp. 100, 219, 220, 221, 225, and 226, with permission from Key Curriculum Press).

The activity will address the question “When are two triangles congruent?” Participants will complete the activity in groups. Have each member within a group complete all the constructions. Groups will need white paper, compasses, rulers, protractors, and/or spaghetti to complete the activity. Then lead a whole-group discussion to formalize the congruence theorems.

Follow the directions to discover the circumstances under which two triangles are congruent. Figures may be constructed using compass, ruler, protractor, or spaghetti.

1. Construct a triangle on paper from the three measurements given. Cut strips of paper to the appropriate lengths or use spaghetti cut to the appropriate lengths. Be sure you match up the endpoints labeled with the same letter.

$$AC = 4 \text{ in.}$$

$$BC = 5 \text{ in.}$$

$$AB = 7 \text{ in.}$$

Compare your triangle with the triangles made by others in your group. (One way to compare them is to place the triangles on top of each other to see if they coincide.) Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Remind participants that they cannot arbitrarily select any three lengths for the sides of a triangle to be assured that those sides will form a triangle.

Side-Side-Side (SSS):

If the three sides of one triangle are congruent to the three sides of another triangle, what can we conclude?

The triangles are congruent. This is known as the Side-Side-Side Triangle Congruence Theorem (SSS).

2. Construct a triangle from the measurements given. Be sure to match up the endpoints labeled with the same letter.

$$DE = 6 \text{ in.}$$

$$DF = 5 \text{ in.}$$

$$m\angle D = 20^\circ$$

Compare your triangle with the triangles made by others in your group. (One way to compare them is to place the triangles on top of each other to see if they coincide.) Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Side-Angle-Side (SAS):

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, what can we conclude?

The triangles are congruent. This is known as the Side-Angle-Side Triangle Congruence Theorem (SAS).

3. Construct a triangle from the three measurements given. Be sure that the side is included between the given angles.

$$MT = 8 \text{ in.}$$

$$m\angle M = 30^\circ$$

$$m\angle T = 50^\circ$$

Compare your triangle with the triangles made by others in your group. Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Angle-Side-Angle (ASA):

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, what can we conclude?

The triangles are congruent. This is known as the Angle-Side-Angle Triangle Congruence Theorem (ASA).

4. Construct a triangle from the three measurements given.

$$ST = 6 \text{ in.}$$

$$TU = 3 \text{ in.}$$

$$m\angle S = 20^\circ$$

Compare your triangle with the triangles made by others in your group. Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Side-Side-Angle (SSA):

If two sides and a non-included angle of one triangle are congruent to two sides and a non-included angle of another triangle, what can we conclude?

The triangles are not always congruent, because two different triangles are possible. Therefore, there is no Side-Side-Angle congruence theorem.

5. Construct a triangle from the three measurements given.

$$m\angle M = 50^\circ$$

$$m\angle N = 60^\circ$$

$$m\angle O = 70^\circ$$

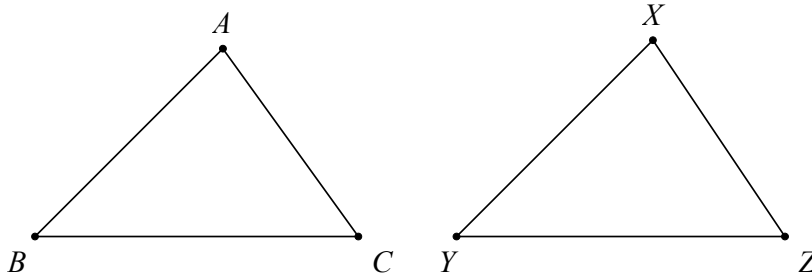
Compare your triangle with the triangles made by others in your group. (One way to compare them is to place the triangles on top of each other to see if they coincide.) Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Angle-Angle-Angle (AAA):

If three angles of one triangle are congruent to three angles of another, what can we conclude?

The triangles are not always congruent, because there are an infinite number of possible triangles of different side lengths with those angle measures. Therefore, there is no Angle-Angle-Angle congruence theorem.

6. In $\triangle ABC$ and $\triangle XYZ$ given below, label $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\overline{BC} \cong \overline{YZ}$. Is $\triangle ABC \cong \triangle XYZ$? Explain your answer.



If two angles in one triangle are congruent to two angles in another, then the third pair of angles are congruent, i.e. $\angle C \cong \angle Z$. So we now have two angles and the included side of one triangle congruent to two angles and the included side of another. By the ASA Congruence Theorem, $\triangle ABC \cong \triangle XYZ$. The AAS Congruence Theorem follows directly from the ASA Congruence Theorem.

Angle-Angle-Side Triangle Congruence Theorem (AAS):

If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, what can we conclude?

The triangles are congruent. This is known as the Angle-Angle-Side Triangle Congruence Theorem (AAS).

Success with this activity indicates that participants are working initially at the Descriptive Level, as they use inductive methods to determine triangle congruence. Participants approach the Deductive Level in 6, when they relate properties from previously determined combinations of properties.

Inductive Triangle Congruences

Follow the directions to discover the circumstances under which two triangles are congruent. Figures may be constructed using compass, ruler, protractor, or spaghetti.

1. Construct a triangle on paper from the three measurements given. Cut strips of paper to the appropriate lengths or use spaghetti cut to the appropriate lengths. Be sure you match up the endpoints labeled with the same letter.

$$AC = 4 \text{ in.}$$

$$BC = 5 \text{ in.}$$

$$AB = 7 \text{ in.}$$

Compare your triangle with the triangles made by others in your group. (One way to compare them is to place the triangles on top of each other to see if they coincide.) Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Side-Side-Side (SSS):

If the three sides of one triangle are congruent to the three sides of another triangle, what can we conclude?

2. Construct a triangle from the measurements given. Be sure to match up the endpoints labeled with the same letter.

$$DE = 6 \text{ in.}$$

$$DF = 5 \text{ in.}$$

$$m\angle D = 20^\circ$$

Compare your triangle with the triangles made by others in your group. (One way to compare them is to place the triangles on top of each other to see if they coincide.) Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Side-Angle-Side (SAS):

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, what can you conclude?

3. Construct a triangle from the three measurements given. Be sure that the side is included between the given angles.

$$MT = 8 \text{ in.}$$

$$m\angle M = 30^\circ$$

$$m\angle T = 50^\circ$$

Compare your triangle with the triangles made by others in your group. Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Angle-Side-Angle (ASA):

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, what can you conclude?

4. Construct a triangle from the three measurements given.

$$ST = 6 \text{ in.}$$

$$TU = 3 \text{ in.}$$

$$m\angle S = 20^\circ$$

Compare your triangle with the triangles made by others in your group. Is it possible to construct different triangles from the given information, or will all the triangles be congruent?

Side-Side-Angle (SSA):

If two sides and a non-included angle of one triangle are congruent to two sides and a non-included angle of another triangle, what can we conclude?

5. Construct a triangle from the three measurements given.

$$m\angle M = 50^\circ$$

$$m\angle N = 60^\circ$$

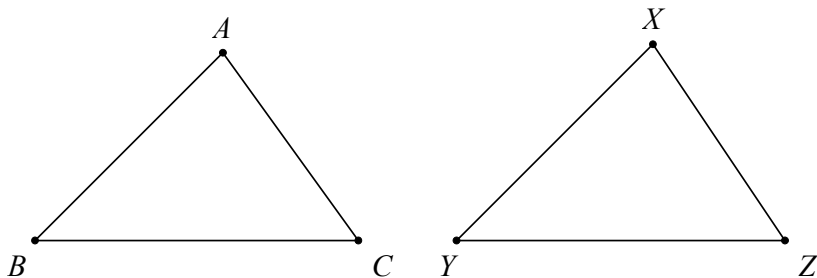
$$m\angle O = 70^\circ$$

Compare your triangle with the triangles made by others in your group. (One way to compare them is to place the triangles on top of each other to see if they coincide.) Is it possible to construct different triangles from the given information or will all the triangles be congruent?

Angle-Angle-Angle (AAA):

If three angles of one triangle are congruent to three angles of another, what can you conclude?

6. In $\triangle ABC$ and $\triangle XYZ$ given below, label $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\overline{BC} \cong \overline{YZ}$. Is $\triangle ABC \cong \triangle XYZ$? Explain your answer.



Angle-Angle-Side (AAS):

If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, what can we conclude?

Deductive Triangle Congruence

Overview: Participants use the triangle congruence theorems to prove that given triangles are congruent.

Objective: **TE_xES Mathematics Competencies**
III.012.A. The beginning teacher understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
III.012.D. The beginning teacher uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.
V.018.A. The beginning teacher understands the nature of proof, including indirect proof, in mathematics.
V.018.B. The beginning teacher applies correct mathematical reasoning to derive valid conclusions from a set of premises.
V.018.C. The beginning teacher uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS

b.1.A. The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems.
b.2.B. The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.
b.3.B. The student constructs and justifies statements about geometric figures and their properties.
b.3.C. The student demonstrates what it means to prove mathematically that statements are true.
b.3.E. The student uses deductive reasoning to prove a statement.
e.3.B. The student justifies and applies triangle congruence relationships.

Background: Participants need to know the SSS, SAS, AAS, and ASA triangle congruence theorems.

Materials:

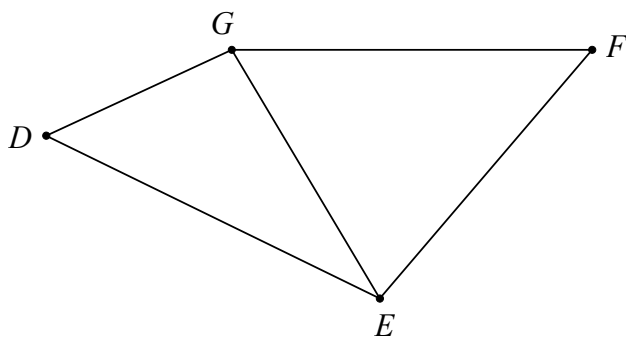
New Terms: reflexive property

Procedures:

If necessary, review the SSS, SAS, ASA, and AAS triangle congruence theorems. In the activity, participants will determine if two triangles are congruent, and if they are congruent, they will deductively prove congruence.

Participants will need to use the reflexive property in this activity. The *reflexive property* states that a number is equal to itself. It will be used to describe when two figures share a common side.

For example, in $\triangle DEG$ and $\triangle EFG$ shown below, $\overline{EG} \cong \overline{EG}$ by the reflexive property.



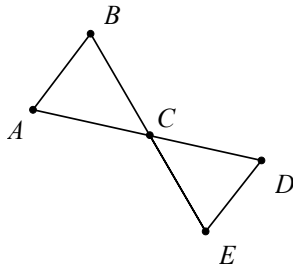
Remind participants to add the term reflexive property to their glossaries.

The following problems are taken from *Discovering Geometry: An Investigative Approach*, 3rd Edition, © 2003, pp. 227-228, 231 with permission from Key Curriculum Press.

Discuss the activity sheet with participants. List all the facts that may help prove that the two triangles in 1 are congruent. Participants work together to complete 2–14.

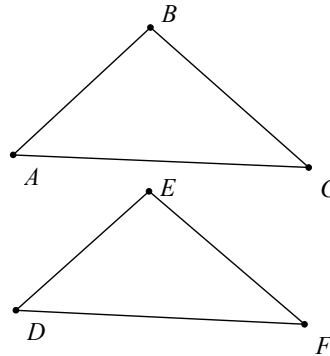
Determine if each pair of triangles below is congruent. List facts about the triangles that help in your determination and mark them in the figures. If they are congruent, state the congruence theorem used to prove the two triangles congruent.

1.



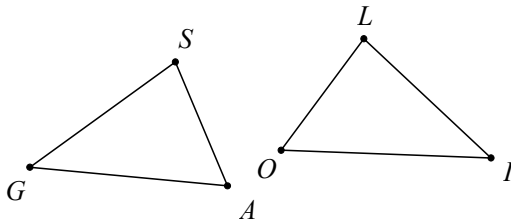
$\angle A \cong \angle D$ (Given)
 $\overline{AC} \cong \overline{CD}$ (Given)
 $\angle ACB \cong \angle DCE$ (Vertical angles are congruent.)
 $\triangle ACB \cong \triangle DCE$ (ASA)

2.



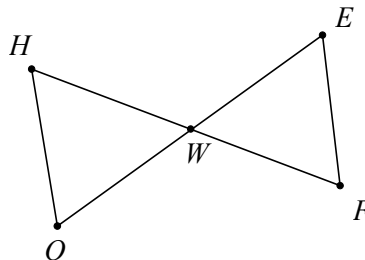
$\angle F \cong \angle A$ (Given)
 $\angle D \cong \angle C$ (Given)
 $\overline{DE} \cong \overline{EC}$ (Given)
 $\triangle ABC \cong \triangle FED$ (ASA)

3.



$\overline{AS} \cong \overline{LO}$ (Given)
 $\angle A \cong \angle O$ (Given)
 $\angle G \cong \angle I$ (Given)
 $\triangle AGS \cong \triangle OIL$ (AAS)

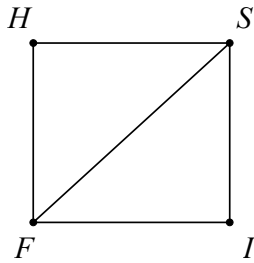
4.



$\overline{HW} \cong \overline{FW}$ (Given)
 $\angle HWO \cong \angle EWF$ (Vertical angles are congruent.)

Not enough information is given to prove that the two triangles are congruent.

5.



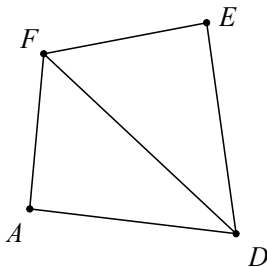
$$\angle HFS \cong \angle SFI \quad (\text{Given})$$

$$\overline{HS} \cong \overline{SI} \quad (\text{Given})$$

$$\overline{FS} \cong \overline{FS} \quad (\text{Reflexive property})$$

Not enough information is given to prove that the two triangles are congruent.

7.



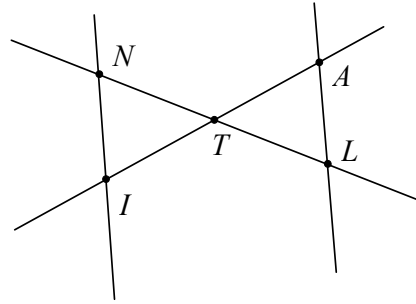
$$\overline{AD} \cong \overline{ED}, \quad (\text{Given})$$

$$\overline{AF} \cong \overline{EF}$$

$$\overline{DF} \cong \overline{DF} \quad (\text{Reflexive property})$$

$$\triangle FAD \cong \triangle FED \quad (\text{SSS})$$

6.



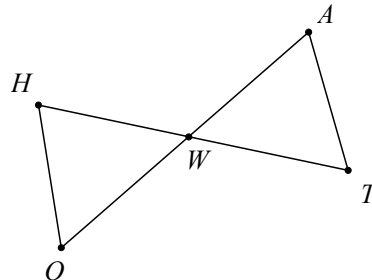
$$\overline{IN} \parallel \overline{AL} \quad (\text{Given})$$

$$\angle ITN \cong \angle ATL \quad (\text{Vertical angles are congruent.})$$

$$\begin{aligned} \angle TAL \cong \angle TIN, \\ \angle TNI \cong \angle TLA \end{aligned} \quad (\text{If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.})$$

Not enough information is given to prove that the two triangles are congruent.

8.



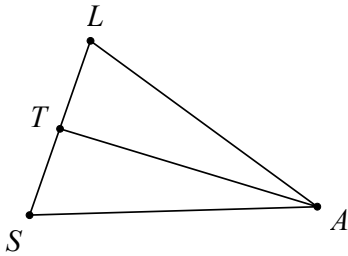
$$\overline{OH} \parallel \overline{AT}, \quad (\text{Given})$$

$$\overline{HW} \cong \overline{WT}$$

$$\begin{aligned} \angle WOH \cong \angle WAT, \\ \angle WHO \cong \angle WTA \end{aligned} \quad (\text{If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.})$$

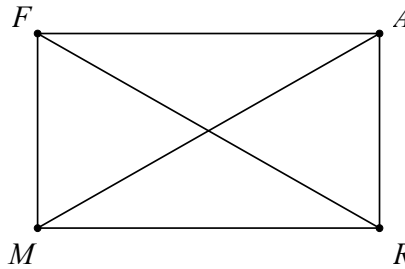
$$\triangle WHO \cong \triangle WTA \quad (\text{ASA})$$

9.



$$\begin{aligned} \angle LAT &\cong \angle TAS && \text{(Given)} \\ \overline{LA} &\cong \overline{AS} && \text{(Given)} \\ \overline{AT} &\cong \overline{AT} && \text{(Reflexive property)} \\ \\ \Delta LAT &\cong \Delta SAT && \text{(SAS)} \end{aligned}$$

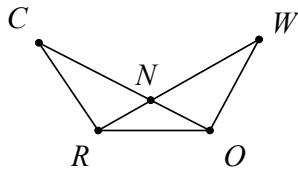
10.



$$\begin{aligned} m\angle FMR &= 90^\circ && \text{(Given)} \\ m\angle ARM &= 90^\circ && \text{(Given)} \\ \overline{FM} &\cong \overline{AR} && \text{(Given)} \\ \\ \overline{MR} &\cong \overline{MR} && \text{(Reflexive property)} \\ \\ \Delta RMF &\cong \Delta MRA && \text{(AAS)} \end{aligned}$$

Use the triangle congruence theorems to answer the questions below. Explain your answers.

11.



Given: $\overline{CN} \cong \overline{WN}$
 $\angle C \cong \angle W$

Is $\overline{RN} \cong \overline{ON}$?

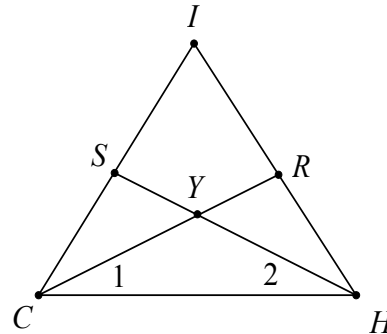
Yes

$$\angle RNC \cong \angle ONW \quad \text{(Vertical angles are congruent.)}$$

$$\Delta CNR \cong \Delta WNO \quad \text{(ASA)}$$

$$\overline{RN} \cong \overline{ON} \quad \text{(Corresponding parts of congruent triangles are congruent.)}$$

12.

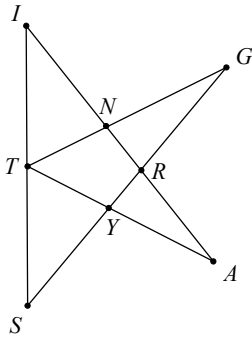


Given: $\overline{CS} \cong \overline{HR}$
 $\angle 1 \cong \angle 2$

Is $\overline{CR} \cong \overline{HS}$?

This cannot be determined. The congruent parts lead to the ambiguous case SSA for ΔSCH and ΔRHC .

13.



Given: $\angle S \cong \angle I$
 $\angle G \cong \angle A$
 T is the midpoint of \overline{SI}

Is $\overline{SG} \cong \overline{IA}$?

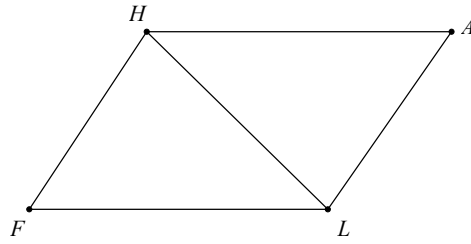
Yes

$\overline{TS} \cong \overline{IT}$ (Definition of midpoint)

$\triangle TSG \cong \triangle TIA$ (AAS)

$\overline{SG} \cong \overline{IA}$ (Corresponding parts of congruent triangles are congruent.)

14.



Given: $HALF$ is a parallelogram.

Is $\overline{HA} \cong \overline{HF}$?

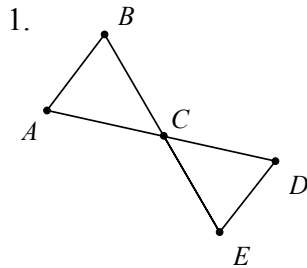
This cannot be determined.

$\triangle HLF \cong \triangle LHA$ by ASA. However, \overline{HA} and \overline{HF} are not corresponding sides of congruent triangles, therefore we know nothing about the relationship of their lengths.

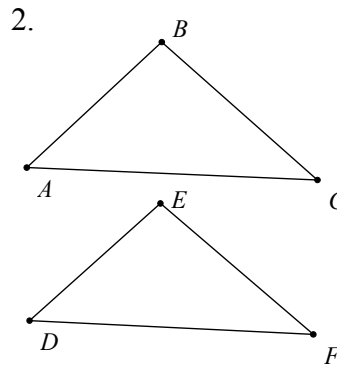
Success with this activity indicates that participants are working at the Deductive Level, because they produce formal deductive arguments.

Deductive Triangle Congruence

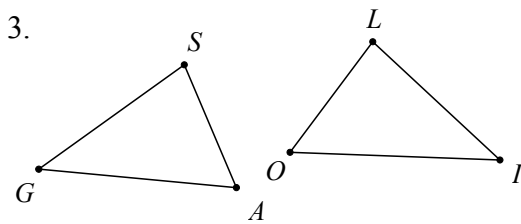
Determine if each pair of triangles below is congruent. List facts about the triangles that help in your determination and mark them in the figures. If they are congruent, state the congruence theorem used to prove the two triangles congruent.



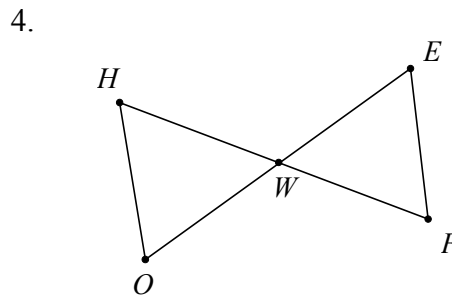
Given: $\angle A \cong \angle D$
 $\overline{AC} \cong \overline{CD}$



Given: $\angle F \cong \angle A$
 $\angle D \cong \angle C$
 $\overline{DF} \cong \overline{AC}$

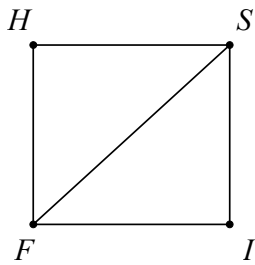


Given: $\overline{AS} \cong \overline{LO}$
 $\angle A \cong \angle O$
 $\angle G \cong \angle I$



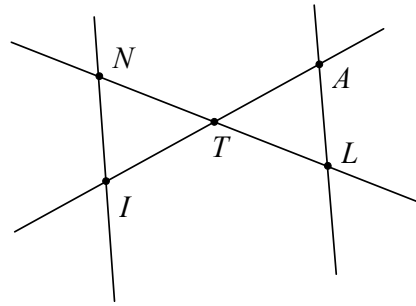
Given: $\overline{HW} \cong \overline{FW}$

5.



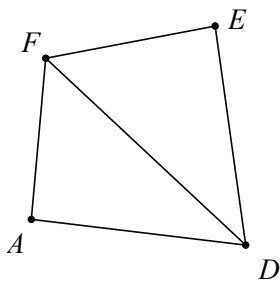
Given: $\angle HFS \cong \angle SFI$
 $\overline{HS} \cong \overline{SI}$

6.



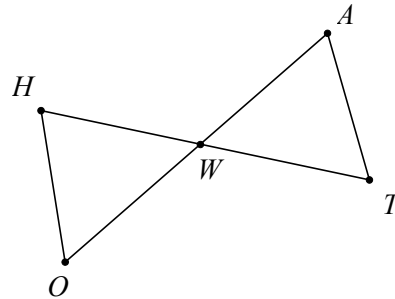
Given: $\overline{IN} \parallel \overline{AL}$

7.



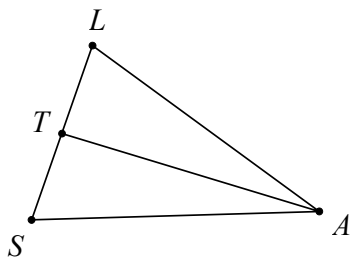
Given: $\overline{AD} \cong \overline{ED}$
 $\overline{AF} \cong \overline{EF}$

8.



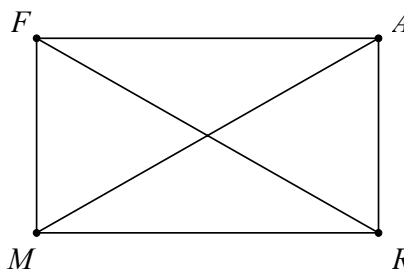
Given: $\overline{OH} \parallel \overline{AT}$
 $\overline{HW} \cong \overline{WT}$

9.



Given: $\angle LAT \cong \angle TAS$
 $\overline{LA} \cong \overline{AS}$

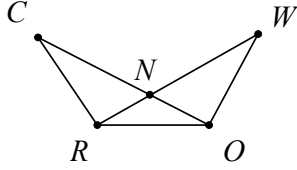
10.



Given: $m\angle FMR = 90^\circ$
 $m\angle ARM = 90^\circ$
 $\overline{FM} \cong \overline{AR}$

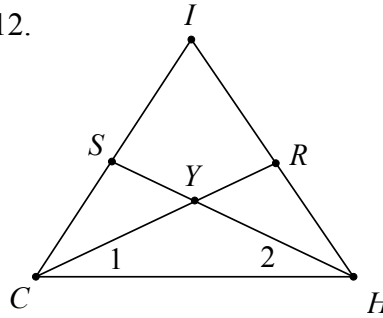
Use the triangle congruence theorems to answer the questions below.
Explain your answers.

11.



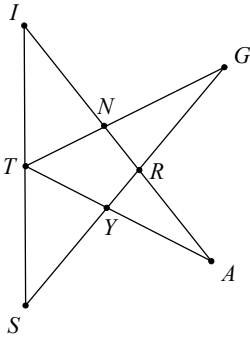
Given: $\overline{CN} \cong \overline{WN}$
 $\angle C \cong \angle W$
 Is $\overline{RN} \cong \overline{ON}$?

12.



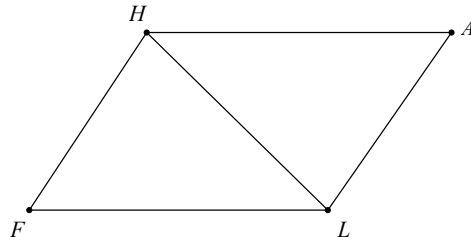
Given: $\overline{CS} \cong \overline{HR}$
 $\angle 1 \cong \angle 2$
 Is $\overline{CR} \cong \overline{HS}$?

13.



Given: $\angle S \cong \angle I$
 $\angle G \cong \angle A$
 T is the midpoint of \overline{SI}
 Is $\overline{SG} \cong \overline{IA}$?

14.



Given: $HALF$ is a parallelogram.
 Is $\overline{HA} \cong \overline{HF}$?

Quadrilateral Proofs

Overview: Using definitions of quadrilaterals and triangle congruence theorems, participants prove properties of quadrilaterals.

Objective: **TE_xES Mathematics Competencies**
III.012.A. The beginning teacher understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
III.012.D. The beginning teacher uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.
V.018.A. The beginning teacher understands the nature of proof, including indirect proof, in mathematics.
V.018.B. The beginning teacher applies correct mathematical reasoning to derive valid conclusions from a set of premises.
V.018.C. The beginning teacher uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS

b.1.A. The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning and theorems.
b.3.B. The student constructs and justifies statements about geometric figures and their properties.
b.3.C. The student demonstrates what it means to prove mathematically that statements are true.
b.3.E. The student uses deductive reasoning to prove a statement.
e.3.B. The student justifies and applies triangle congruence relationships.

Background: Participants need a knowledge of definitions and rules learned in previous units.

Materials: easel paper, colored markers

New Terms:

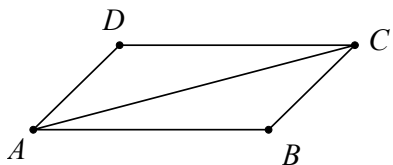
Procedures:

Using the congruence theorems and definitions presented earlier in this module, participants will prove theorems about quadrilaterals. These activities are taken from

Discovering Geometry: An Investigative Approach, 3rd Edition, © 2003, pp. 692-693 with permission from Key Curriculum Press.

Lead a discussion to prove that a diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Put the following on a transparency or on easel paper.



Given: Parallelogram $ABCD$ with diagonal \overline{AC}

Prove: $\triangle ABC \cong \triangle CDA$

Have participants list facts that they observe from the figure.

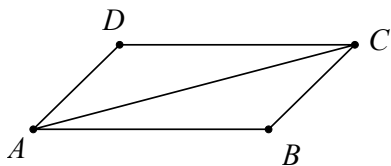
| | |
|---|---|
| Quadrilateral $ABCD$ is a parallelogram | (Given) |
| $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$ | (Opposite sides of a parallelogram are parallel.) |
| $\angle CAB \cong \angle ACD$ and $\angle BCA \cong \angle DAC$ | (If two parallel lines are cut by transversal, then the alternate interior angles are congruent.) |
| $\overline{AC} \cong \overline{AC}$ | (Reflexive property) |
| $\triangle ABC \cong \triangle CDA$ | (ASA) |

This proves that a diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Participants should work in groups to prove 1 – 6 on the activity sheet. They may use the above theorem in their proofs. Give each group easel paper and markers. When all the groups are finished, have each group present on easel paper a different proof to the entire class.

Work with participants in your group to prove the statements below. Before you try to prove each statement, draw a diagram and state both what is given and what you are proving in terms of your diagram.

1. Prove that the opposite sides of a parallelogram are congruent.



Given: Parallelogram $ABCD$ with diagonal \overline{AC}

Prove: $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$

Parallelogram $ABCD$ with diagonal \overline{AC} (Given)

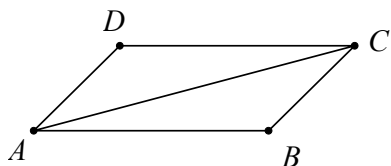
$\triangle ABC \cong \triangle CDA$

(A diagonal of a parallelogram divides the parallelogram into two congruent triangles.)

$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

(Corresponding parts of congruent triangles are congruent.)

2. Prove that the opposite angles of a parallelogram are congruent.



Given: Parallelogram $ABCD$ with diagonal \overline{AC}

Prove: $\angle D \cong \angle B$, $\angle DAB \cong \angle BCD$

Parallelogram $ABCD$ with diagonal \overline{AC} (Given)

$\triangle ABC \cong \triangle CDA$

(A diagonal of a parallelogram divides the parallelogram into two congruent triangles.)

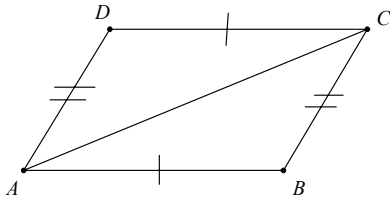
$\angle D \cong \angle B$

(Corresponding parts of congruent triangles are congruent.)

Similarly if we use \overline{BD} as the diagonal instead of \overline{AC} , then the congruent angles would be $\angle DAB$ and $\angle BCD$.

3. State and prove the converse of 1 above.

If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



Given: Quadrilateral $ABCD$, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$

Prove: Quadrilateral $ABCD$ is a parallelogram

Quadrilateral $ABCD$ with a diagonal \overline{AC} (Given)

$\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$ (Given)

$\overline{AC} \cong \overline{AC}$ (Reflexive property)

$\triangle ADC \cong \triangle CBA$ (SSS)

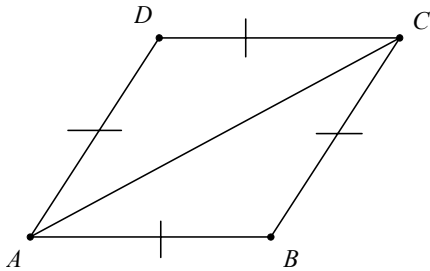
$\angle DCA \cong \angle BAC$ and $\angle DAC \cong \angle ACB$ (Corresponding parts of congruent triangles are congruent.)

$\overline{DC} \parallel \overline{AB}$ and $\overline{DA} \parallel \overline{BC}$ (If two lines are cut by a transversal so that the alternating angles are congruent, then the two lines are parallel.)

Quadrilateral $ABCD$ is a parallelogram (Definition of parallelogram)

A parallel proof could have been constructed using \overline{BD} instead of \overline{AC} as the diagonal of the quadrilateral to prove $\triangle BAD \cong \triangle DCB$.

4. Prove that each diagonal of a rhombus bisects the two opposite angles.



Given: Rhombus $ABCD$ with diagonal \overline{AC}

Prove: $\angle DAC \cong \angle BAC$ and $\angle DCA \cong \angle BCA$

Rhombus $ABCD$ with diagonal \overline{AC} (Given)

$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ (Definition of rhombus)

$\overline{AC} \cong \overline{AC}$ (Reflexive property)

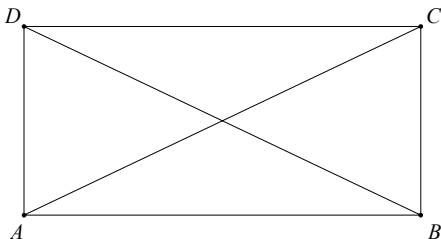
$\triangle ABC \cong \triangle ADC$ (SSS)

$\angle DAC \cong \angle BAC$ and $\angle DCA \cong \angle BCA$ (Corresponding parts of congruent triangles are congruent.)

$\angle DAB$ and $\angle DCB$ are bisected by \overline{AC} (Definition of angle bisector)

A parallel proof could have been constructed using \overline{BD} instead of \overline{AC} as the diagonal of the rhombus to prove that \overline{BD} bisects $\angle DCB$ and $\angle DAB$.

5. Prove that the diagonals of a rectangle are congruent.



Given: Rectangle $ABCD$ with diagonals \overline{AC} and \overline{BD}

Prove: $\overline{AC} \cong \overline{BD}$

$\angle DAB$ and $\angle CBA$ are right angles (Definition of rectangle)
 $\angle ADC$ and $\angle BCD$ are right angles

$\angle DAB \cong \angle CBA$ and $\angle ADC \cong \angle BCD$ (All right angles are congruent.)

$\angle DAB$ and $\angle CBA$ are supplementary angles (Definition of supplementary angles)
 $\angle ADC$ and $\angle BCD$ are supplementary angles

$\overline{DA} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$ (If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then the lines are parallel.)

$ABCD$ is a parallelogram (Definition of a parallelogram)

$$\overline{AB} \cong \overline{CD} \text{ and } \overline{AD} \cong \overline{BC}$$

(Opposite sides of a parallelogram are congruent.)

$$\overline{AB} \cong \overline{AB}$$

(Reflexive property)

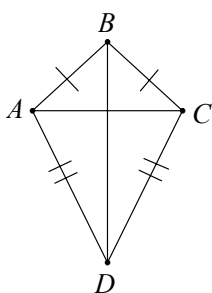
$$\triangle DAB \cong \triangle CBA$$

(SSS)

$$\overline{AC} \cong \overline{BD}$$

(Corresponding sides of congruent triangles are congruent.)

6. Prove that the angles between each pair of congruent sides of a kite are bisected by a diagonal.



Given: Kite $ABCD$ with diagonals \overline{AC} and \overline{BD}

Prove: $\angle ABD \cong \angle CBD$ and $\angle ADB \cong \angle CDB$

Kite $ABCD$ with diagonals \overline{AC} and \overline{BD}

(Given)

$$\overline{AB} \cong \overline{BC} \text{ and } \overline{AD} \cong \overline{CD}$$

(Definition of a kite)

$$\overline{BD} \cong \overline{BD}$$

(Reflexive property)

$$\triangle ABC \cong \triangle CBD$$

(SSS)

$$\angle ABD \cong \angle CBD \text{ and } \angle ADB \cong \angle CDB$$

(Corresponding parts of congruent triangles are congruent.)

Success with this activity indicates that participants are working at the Deductive Level because they develop formal deductive proofs.

Quadrilateral Proofs

Work with participants in your group to prove the statements below. Before you try to prove each statement, draw a diagram, state what is given and what you are proving in terms of your diagram.

1. Prove that the opposite sides of a parallelogram are congruent.

2. Prove that the opposite angles of a parallelogram are congruent.

3. State and prove the converse of 1 above.

4. Prove that each diagonal of a rhombus bisects the two opposite angles.

5. Prove that the diagonals of a rectangle are congruent.

6. Prove that the angles between each pair of congruent sides of a kite are bisected by a diagonal.

Alternate Definitions of Quadrilaterals

Overview: Participants write alternative definitions of quadrilaterals based on their properties.

Objective: **TEExES Mathematics Competencies**
III.012.A. The beginning teacher understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
III.012.D. The beginning teacher uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.
III.013.A. The beginning teacher analyzes the properties of polygons and their components.
III.013.C. The beginning teacher uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).
V.018.B. The beginning teacher applies correct mathematical reasoning to derive valid conclusions from a set of premises.
V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS

b.1.A. The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning and theorems.
b.3.A. The student determines if the converse of a conditional statement is true or false.
b.3.B. The student constructs and justifies statements about geometric figures and their properties.
b.3.C. The student demonstrates what it means to prove mathematically that statements are true.
b.3.E. The student uses deductive reasoning to prove a statement.

Background: Participants should be familiar with definitions and properties of quadrilaterals, conditional statements, and biconditional statements.

Materials: easel paper, colored markers

New Terms:

Procedures:

We have an exhaustive list of the properties for each quadrilateral. This exhaustive list defines the quadrilateral. In this activity we will identify the properties that are sufficient

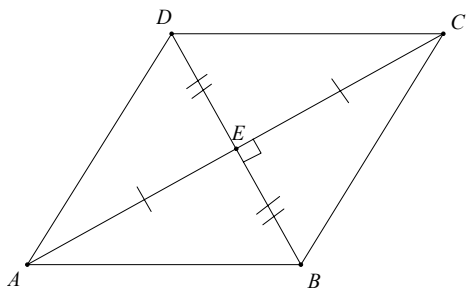
to alternately define the particular quadrilateral. There are many ways of combining properties to sufficiently define a particular quadrilateral.

Work through the following example with the group.

Consider the rhombus:

1. State the basic definition of the rhombus as a conditional statement.
If a quadrilateral is a rhombus, then it has four congruent sides.
2. List properties of the rhombus as conditional statements.
 - *If a quadrilateral is a rhombus, then its opposite sides are parallel.*
 - *If a quadrilateral is a rhombus, then its opposite sides are congruent.*
 - *If a quadrilateral is a rhombus, then its opposite angles are congruent.*
 - *If a quadrilateral is a rhombus, then its consecutive angles are supplementary.*
 - *If a quadrilateral is a rhombus, then its diagonals bisect the vertex angles.*
 - *If a quadrilateral is a rhombus, then its diagonals are perpendicular to each other.*
 - *If a quadrilateral is a rhombus, then its diagonals bisect each other.*
3. Select no more than two properties which can be combined to sufficiently define the rhombus alternately.
In this example we will select perpendicular diagonals and bisecting diagonals.
4. Write a proof to show that the properties selected in 3 sufficiently define the rhombus.

Diagram:



Given: *Quadrilateral ABCD with diagonals \overline{AC} and \overline{BD} intersecting at point E,*
 $\overline{AC} \perp \overline{BD}$, $\overline{EA} \cong \overline{EC}$ and $\overline{EB} \cong \overline{ED}$

Prove: *Quadrilateral ABCD is a rhombus.*

$$\overline{AC} \perp \overline{BD}$$

(Given)

$$\angle AEB \cong \angle BEC \cong \angle CED \cong \angle DEA$$

(Perpendicular lines intersect to form four congruent right angles.)

$$\overline{EA} \cong \overline{EC} \text{ and } \overline{EB} \cong \overline{ED}$$

(Given)

$$\triangle AEB \cong \triangle AED \cong \triangle CED \cong \triangle CEB \quad (SAS)$$

$$\overline{AB} \cong \overline{DA} \cong \overline{CD} \cong \overline{BC} \quad (\text{Corresponding sides of congruent triangles are congruent.})$$

Quadrilateral ABCD is a rhombus (Definition of a rhombus)

5. If the given properties selected do provide an alternate definition, then rewrite the alternate definition as a biconditional statement, using “if and only if” which implies that both the conditional and its converse are true.

A quadrilateral is a rhombus if and only if the diagonals are perpendicular to each other and bisect each other.

Participants now work in groups, each group focusing on a different quadrilateral from the activity sheet. Provide each group with easel paper and markers. Participants develop an alternate definition for their quadrilateral following the method used above. Each group presents its work on easel paper to the entire group.

The definitions and properties are listed below. Since there are a variety of ways to combine properties to find alternate definitions, answers may vary. The group which works on the rhombus should combine different pairs of properties from those used in the example.

Square

1. State the basic definition of the square as a conditional statement.
If a quadrilateral is a square, then it has four congruent sides and four right angles.
2. List properties of the square as conditional statements.
 - *If a quadrilateral is a square, then its opposite sides are parallel.*
 - *If a quadrilateral is a square, then its sides are congruent.*
 - *If a quadrilateral is a square, then its consecutive sides are perpendicular.*
 - *If a quadrilateral is a square, then its diagonals are congruent to one another.*
 - *If a quadrilateral is a square, then its diagonal bisect each other at right angles.*
 - *If a quadrilateral is a square, then its diagonals bisect the vertex angles.*
 - *If a quadrilateral is a square, then its angles are congruent right angles.*
 - *If a quadrilateral is a square, then its consecutive angles are congruent and supplementary.*
 - *If a quadrilateral is a square, then its opposite angles are congruent and supplementary.*

Kite

1. State the basic definition of the kite as a conditional statement.
If a quadrilateral is a kite, then it has two distinct pairs of consecutive congruent sides.

2. List properties of the kite as conditional statements.

- *If a quadrilateral is a kite, then its opposite sides are not congruent.*
- *If a quadrilateral is a kite, then its diagonals are perpendicular to each other.*
- *If a quadrilateral is a kite, then it has one pair of congruent opposite angles.*
- *If a quadrilateral is a kite, then only one of its diagonals is bisected by the other diagonal, and the bisected diagonal has its endpoints on the congruent angles' vertex.*

Parallelogram

1. State the basic definition of the parallelogram as a conditional statement.

If a quadrilateral is a parallelogram, then it has two pairs of parallel sides.

2. List properties of the parallelogram as conditional statements.

- *If a quadrilateral is a parallelogram, then its opposite sides are congruent.*
- *If a quadrilateral is a parallelogram, then its opposite angles are congruent.*
- *If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.*
- *If a quadrilateral is a parallelogram, then its diagonals bisect one another.*

Rectangle

1. State the basic definition of the rectangle as a conditional statement.

If a quadrilateral is a rectangle, then it has four right angles.

2. List properties of the rectangle as conditional statements.

- *If a quadrilateral is a rectangle, then its opposite sides are congruent.*
- *If a quadrilateral is a rectangle, then its opposite sides are parallel.*
- *If a quadrilateral is a rectangle, then its opposite angles are congruent.*
- *If a quadrilateral is a rectangle, then its opposite angles are supplementary.*
- *If a quadrilateral is a rectangle, then its consecutive angles are congruent.*
- *If a quadrilateral is a rectangle, then its consecutive angles are supplementary.*
- *If a quadrilateral is a rectangle, then its diagonals are congruent.*
- *If a quadrilateral is a rectangle, then its diagonals bisect one another.*

Ask groups to present their work to close the activity.

Success with this activity indicates that participants are working at the Deductive Level, because they created deductive proofs.

4. Write a proof to show that the properties selected in 3 sufficiently define the quadrilateral.

Diagram:

Given:

Prove:

Proof:

5. If the given properties selected do provide an alternate definition, then rewrite the alternate definition as a biconditional statement, using “if and only if” which implies that both the conditional and its converse are true.

Circle Proofs

Overview: Participants prove theorems about inscribed angles.

Objective: **TEXES Mathematics Competencies**

III.012.A. The beginning teacher understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).

III.012.D. The beginning teacher uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.

III.013.B. The beginning teacher analyzes the properties of circles and the lines that intersect them.

III.013.C. The beginning teacher uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).

V.018.A. The beginning teacher understand the nature of proof, including indirect proof, in mathematics.

V.018.B. The beginning teacher applies correct mathematical reasoning to derive valid conclusions from a set of premises.

V.018.C. The beginning teacher uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.

V.018.D. The beginning teacher uses formal and informal reasoning to justify mathematical ideas.

Geometry TEKS

b.1.A. The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning and theorems.

b.2.B. The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

b.3.B. The student constructs and justifies statements about geometric figures and their properties.

b.3.C. The student demonstrates what it means to prove mathematically that statements are true.

b.3.E. The student uses deductive reasoning to prove a statement.

e.2.C. Based on explorations and using concrete models, the student formulates and tests conjectures about the properties and attributes of circles and the lines that intersect them.

Background: Participants need to be familiar with exterior-interior angle relationships in triangles.

Materials:

New Terms: intercepted arc

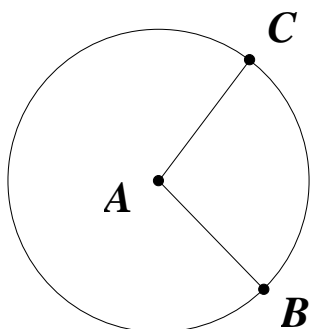
Procedures:

We formally introduce circle properties in a later unit, but we will explore them in this unit with theorems taken from *Discovering Geometry: An Investigative Approach*, 3rd Edition, © 2003, pp. 325-327 with permission from Key Curriculum Press.

Participants should recall the definitions of central angle and inscribed angle. A central angle has its vertex at the center of the circle. An inscribed angle has its vertex on the circle and its sides are chords.

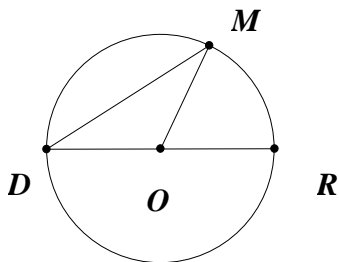
We define *intercepted arc* of a circle as the part of a circle whose endpoints are the points where the segments of a central angle intersect the circle.

In the following proofs, we use the fact that the measure of a central angle is equal to the measure of its intercepted arc. In the diagram below $m\angle BAC = m\widehat{BC}$, i.e., if $m\angle BAC = 25^\circ$, then $m\widehat{BC} = 25^\circ$ and visa versa.



Remind participants to add the term intercepted arc to their glossaries.

1. Show that the measure of an inscribed angle ($\angle MDR$) in a circle equals half the measure of its central angle ($\angle MOR$) that intercepts the same arc (\widehat{RM}) when a side of the angle, \overline{DR} , passes through the center of the circle.



Given: Circle O with inscribed $\angle MDR$ on diameter \overline{DR}

Prove: $m \angle MDR = \frac{1}{2} m \angle MOR$

$$m \angle MOR = m \widehat{MR}$$

(The measure of a central angle is equal to the measure of its intercepted arc.)

$$m \angle MOR = m \angle MDO + m \angle DMO$$

(The measure of the remote exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles of the triangle.)

$$\overline{DO} \cong \overline{MO}$$

(Radii of the same circle are congruent.)

$\triangle DMO$ is isosceles

(Definition of isosceles triangle.)

$$m \angle MDO = m \angle DMO$$

(Base angles of an isosceles triangle are congruent.)

$$m \angle MOR = m \angle MDO + m \angle DMO$$

(Substitution)

$$m \angle MOR = 2 m \angle MDO$$

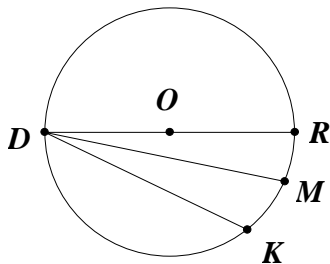
(Combine like terms)

$$m \angle MDO = \frac{1}{2} m \angle MOR$$

(Divide by 2)

This proves that the measure of the inscribed angle in a circle equals half the measure of its central angle that intercepts the same arc.

2. Show that the measure of an inscribed angle in a circle equals half the measure of its intercepted arc when the center of the circle is outside the angle.



Given: Circle O with inscribed $\angle MDK$ on one side of diameter \overline{DR}

Prove: $m\angle MDK = \frac{1}{2} \widehat{KM}$

$$m\angle KDR = m\angle MDR + m\angle MDK \quad (\text{Angle Addition})$$

$$m\angle MDK = m\angle KDR - m\angle MDR \quad (\text{Subtract } m\angle MDR)$$

$$m\angle MDR = \frac{1}{2} \widehat{MR} \quad \text{and} \quad m\angle KDR = \frac{1}{2} \widehat{KR} \quad (\text{We proved that the measure of an inscribed angle in a circle equals half the measure of its central angle when a side of the angle passes through the center of the circle.})$$

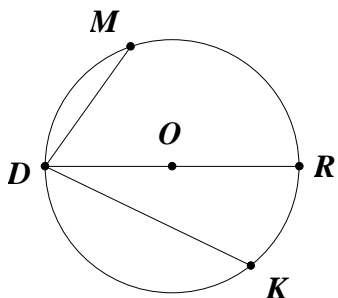
$$m\widehat{KR} = m\widehat{MR} + m\widehat{KM} \quad (\text{Arc Addition})$$

$$m\angle MDK = \frac{1}{2} (\widehat{MR} + \widehat{KM}) - \frac{1}{2} \widehat{MR} \quad (\text{Substitution})$$

$$m\angle MDK = \frac{1}{2} \widehat{KM} \quad (\text{Simplify})$$

Therefore the measure of an inscribed angle in a circle equals half the measure of its intercepted arc when the center of the circle is outside the angle.

3. Show that the measure of an inscribed angle in a circle equals half the measure of its intercepted arc when the center of the circle is inside the angle.



Given: Circle O with inscribed angle $\angle MDK$

Prove: $m\angle MDK = \frac{1}{2} m\widehat{MRK}$

$$m\angle MDK = m\angle MDR + m\angle RDK \quad (\text{Angle Addition})$$

$$m\angle MDR = \frac{1}{2} m\widehat{MR} \quad \text{and} \quad m\angle RDK = \frac{1}{2} m\widehat{RK} \quad (\text{The measure of an inscribed angle})$$

in a circle equals half the measure of its central angle when a side of the angle passes through the center of the circle.)

$$m\angle MDK = \frac{1}{2}m\widehat{MR} + \frac{1}{2}m\widehat{RK} \quad (\text{Substitution})$$

$$m\angle MDK = \frac{1}{2}(m\widehat{MR} + m\widehat{RK}) \quad (\text{Simplify})$$

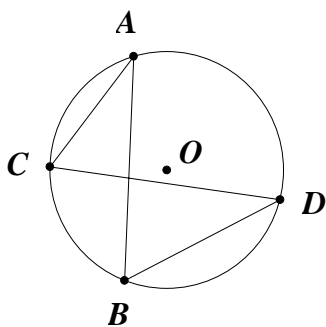
$$m\widehat{MR} + m\widehat{RK} = m\widehat{MRK} \quad (\text{Arc Addition})$$

$$m\angle MDK = \frac{1}{2}m\widehat{MRK} \quad (\text{Substitution})$$

Therefore the measure of an inscribed angle in a circle equals half the measure of its intercepted arc when the center of the circle is inside the angle.

We have proved all three cases. Therefore we can simply state the theorem as the measure of an inscribed angle in a circle equals half the measure of its intercepted arc.

4. Show that inscribed angles that intercept the same arc are congruent.



Given: Circle O with $\angle CAB$ and $\angle BDC$ inscribed in \widehat{BC}

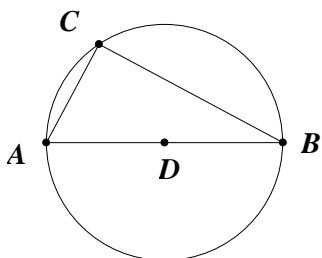
Prove: $m\angle CAB = m\angle BDC$

$$m\angle CAB = \frac{1}{2}\widehat{BC} \text{ and } m\angle BDC = \frac{1}{2}\widehat{BC} \quad (\text{The measure of an inscribed angle in a circle equals half the measure of its intercepted arc.})$$

$$m\angle CAB = m\angle BDC \quad (\text{Substitution})$$

Therefore inscribed angles that intercept the same arc are congruent.

5. Show that the angles inscribed in a semicircle are right angles.



Given: $\angle ACB$ is an inscribed angle, \overline{ADB} is a diameter

Prove: $m\angle ACD = 90^\circ$

$\angle ACB$ is an inscribed angle (Given)

\overline{ADB} is a diameter (Given)

$m\angle ADB = 180^\circ$ (A straight angle measures 180° .)

$m\angle ACD = \frac{1}{2}m\angle ADB$ (The measure of an inscribed angle in a circle equals half the measure of its intercepted arc.)

$m\angle ACD = 90^\circ$ (Substitution)

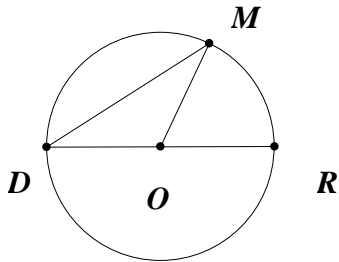
Therefore, the measure of an inscribed angle in a semicircle is $\frac{1}{2}(180^\circ)$ or 90° .

Success in this activity indicates that participants are at the Deductive Level, because they formally develop deductive proofs.

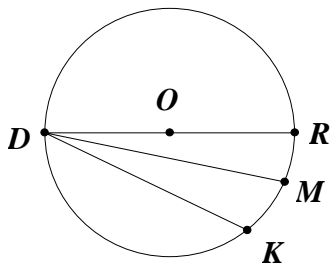
Circle Proofs

Write the given statements and those that are to be proved. Then write the proof itself.

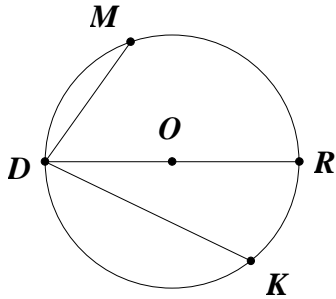
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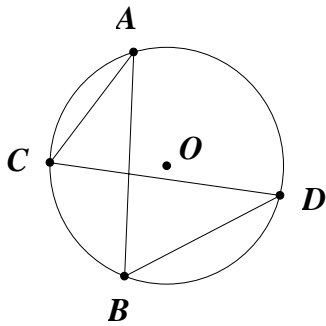
2. Show that the measure of an inscribed angle in a circle equals half the measure of its intercepted arc when the center of the circle is outside the angle.



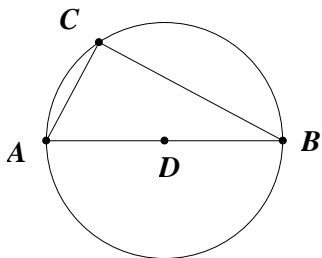
3. Show that the measure of an inscribed angle in a circle equals half the measure of its intercepted arc when the center of the circle is inside the angle.



4. Show that inscribed angles that intercept the same arc are congruent.



5. Show that the angles inscribed in a semicircle are right angles.



References and Additional Resources

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