

Texas Examination of Educator Standards Teacher Competencies for Mathematics (8-12)

DOMAIN I-NUMBER CONCEPTS

Competency 001

The teacher understands the real number system and its structure, operations, algorithms, and representations.

The beginning teacher:

- A. Understands the concepts of place value, number base, and decimal representations of real numbers.
- B. Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).
- C. Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).
- D. Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- E. Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms, and real numbers.
- F. Uses real numbers to model and solve a variety of problems.
- G. Uses deductive reasoning to simplify and justify algebraic processes.
- H. Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

Competency 002

The teacher understands the complex number system and its structure, operations, algorithms, and representations.

The beginning teacher:

- A. Demonstrates how some problems that have no solution in the real number system have solutions in the complex number system.
- B. Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).
- C. Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).

- D. Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.
- E. Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

Competency 003

The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.

The beginning teacher:

- A. Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.
- B. Applies number theory concepts and principles to justify and prove number relationships.
- C. Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, non-commutative operations).
- D. Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.
- E. Applies counting technique such as permutations and combinations to quantify situations and solve problems.
- F. Uses estimation techniques to solve problems and judges the reasonableness of solutions.

DOMAIN II-PATTERNS AND ALGEBRA**Competency 004**

The teacher uses patterns to model and solve problems and formulate conjectures.

The beginning teacher:

- A. Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.
- B. Uses methods of recursion and iteration to model and solve problems.
- C. Uses the principle of mathematical induction.
- D. Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.
- E. Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound, and continuous interest rates; annuities).

Competency 005

The teacher understands attributes of functions, relations, and their graphs.

The beginning teacher:

- A. Understands when a relation is a function.
- B. Identifies the mathematical domain and the range of functions and relations and determines reasonable domains for given situations.
- C. Understand that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).
- D. Identifies and analyzes even and odd functions, one-to-one functions, inverse functions, and their graphs.
- E. Applies basic transformations [e.g., $kf(x)$, $f(x) + k$, $f(x-k)$, $f(kx)$, $|f(x)|$] to a parent function, f , and describes the effects on the graph of $y = f(x)$.
- F. Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations, and describes results symbolically and graphically.
- G. Uses graphs of functions to formulate conjectures of identities [e.g., $y = x^2 - 1$ and $y = (x-1)(x+1)$, $y = \log x^3$ and $y = 3 \log x$, $y = \sin(x + \frac{\pi}{2})$ and $y = \cos x$].

Competency 006

The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The beginning teacher:

- A. Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.
- B. Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and y-intercept).
- C. Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems.
- D. Analyzes the zeros (real and complex) of quadratic functions.
- E. Makes connections between the $y = ax^2 + bx + c$ and the $y = a(x-h)^2 + k$ representations of a quadratic function and its graph.
- F. Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).

- G. Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

Competency 007

The teacher understands polynomial, rational, radical, absolute value, and piecewise functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value, and piecewise functions.
- B. Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value, and piecewise functions.
- C. Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or not differentiable) of a function, the graph of a function, and the function's symbolic representation.
- D. Analyzes functions in terms of vertical, horizontal, and slant asymptotes.
- E. Analyzes and applies the relationship between inverse variation and rational functions.
- F. Solves equations and inequalities involving polynomial, rational, radical, absolute value, and piecewise functions using a variety of methods (e.g., tables, algebraic methods, graphs, use of graphing calculator), and evaluates the reasonableness of solutions.
- G. Models situations using polynomial, rational, radical, absolute value, and piecewise functions and solves problems using a variety of methods, including technology.

Competency 008

The teacher understands exponential and logarithmic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.
- B. Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function, and the function's symbolic representation.

- C. Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.
- D. Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities, and systems involving exponential and logarithmic functions.
- E. Models and solves problems involving exponential growth and decay.
- F. Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.
- G. Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance. (e.g., compound interest).
- H. Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e., $f'(x) = k f(x)$].

Competency 009

The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The beginning teacher:

- A. Analyzes the relationship among the unit circle in the coordinate plane, circular functions, and the trigonometric functions.
- B. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.
- C. Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of a trigonometric function, the graph of the function, and the function's symbolic representation.
- D. Understands the relationship between trigonometric functions and their inverses and uses these relationships to solve problems.
- E. Uses trigonometric identities to simplify expressions and solve equations.
- F. Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.
- G. Uses graphing calculators to analyze and solve problems involving trigonometric functions.

Competency 010

The teacher understands and solves problems using differential and integral calculus.

The beginning teacher:

- A. Understands the concept of limit and the relationship between limits and continuity.
- B. Relates the concept of average rate of change to the slope of the secant line and the concept of instantaneous rate of change to the slope of the tangent line.
- C. Uses the first and second derivations to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).
- D. Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.
- E. Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.
- F. Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

DOMAIN III-GEOMETRY AND MEASUREMENT**Competency 011**

The teacher understands measurement as a process.

The beginning teacher:

- A. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- B. Applies to formulas for perimeter, area, surface area, and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
- C. Recognizes the effects on length, area, or volume when the linear dimensions of plane figures or solids are changed.
- D. Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.
- E. Relates the concept of area under a curve to the limit of a Riemann sum.

- F. Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane and measurements associated with curves, surfaces, and regions in three-space.

Competency 012

The teacher understands geometries, in particular Euclidean geometry, as axiomatic systems.

The beginning teacher:

- A. Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
- B. Uses properties of points, lines, planes, angles, lengths, and distances to solve problems.
- C. Applies the properties of parallel and perpendicular lines to solve problems.
- D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.
- E. Describes and justifies geometric constructions made using compass and straightedge, reflection devices, and other appropriate technologies.
- F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.
- G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (e.g., hyperbolic and elliptic geometry).

Competency 013

The teacher understands the results, uses, and applications of Euclidean geometry.

The beginning teacher:

- A. Analyzes the properties of polygons and their components.
- B. Analyzes the properties of circles and the lines that intersect them.
- C. Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).
- D. Computes the perimeter, area, and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).
- E. Analyzes cross-sections and nets of three-dimensional shapes.
- F. Uses top, front, side, and corner views of three-dimensional shapes to create complete representations and solve problems.

- G. Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life.

Competency 014

The teacher understands coordinate, transformational, and vector geometry and their connections.

The beginning teacher:

- A. Identifies transformations (i.e., reflections, translations, glide-reflections, rotations, dilations) and explores their properties.
- B. Uses the properties of transformations and their compositions to solve problems.
- C. Uses transformations to explore and describe reflectional, rotational, and translational symmetry.
- D. Applies transformations in the coordinate plane.
- E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity, and distance to explore properties of geometric figures and solve problems in the coordinate plane.
- F. Uses coordinate geometry to derive and explore the equations, properties, and applications of conic sections. (i.e., lines, circles, hyperbolas, ellipses, parabolas).
- G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.
- H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

DOMAIN IV-PROBABILITY AND STATISTICS**Competency 015**

The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns, and describe departures from patterns.

The beginning teacher:

- A. Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyzes data.
- B. Organizes, displays, and interprets data in a variety of formats (e.g., tables, frequency distributions, scatter plots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).

- C. Applies concepts of center, spread, shape, and skewness to describe a data distribution.
- D. Understands measures of central tendency (i.e., mean, median, mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).
- E. Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.
- F. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers, and measures of central tendency and dispersion.
- G. Supports arguments, makes predictions, and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

Competency 016**The teacher understands concepts and applications of probability.**

The beginning teacher:

- A. Understands how to explore concepts of probability through sampling, experiments, and simulations, and generates and uses probability models to represent situations.
- B. Uses the concepts and principles of probability to describe the outcomes of simple and compound events.
- C. Determines probabilities by constructing sample spaces to model situations.
- D. Solves a variety of probability problems using combinations and permutations.
- E. Solves a variety of probability problems using ratios of areas of geometric regions.
- F. Calculates probabilities using the axioms of probability and related theorems and concepts such as the addition rule, multiplication rule, conditional probability, and independence.
- G. Understands expected value, variance, and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).
- H. Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

Competency 017

The teacher understands the relationship among probability theory, sampling, and statistical inference, and how statistical inference is used in making and evaluating predictions.

The beginning teacher:

- A. Applies knowledge of designing, conducting, analyzing, and interpreting statistical experiments to investigate real-world problems.
- B. Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognize misleading as well as valid uses of statistics.
- C. Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).
- D. Makes inferences about a population using binomial, normal, and geometric distributions.
- E. Describes and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).
- F. Understands how to transform nonlinear data into linear form in order to apply linear regression techniques to develop exponential, logarithmic, and power regression models.
- G. Uses the law of large numbers and the central limit theorem in the process of statistical inference.
- H. Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).
- I. Understands principles of hypotheses testing.

DOMAIN V-MATHEMATICAL PROCESSES AND PERSPECTIVES**Competency 018**

The teacher understands mathematical reasoning and problem solving.

The beginning teacher:

- A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).
- B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).

- C. Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.
- D. Uses formal and informal reasoning to justify mathematical ideas.
- E. Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).
- F. Evaluates how well a mathematical model represents a real-world situation.

Competency 019

The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

The beginning teacher:

- A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of the circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).
- B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music science, social science, business).
- C. Translates mathematical ideas between verbal and symbolic forms.
- D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).
- E. Understands the use of visual media, such as graphs, tables, diagrams, and animations, to communicate mathematical information.
- F. Uses appropriate mathematical terminology to express mathematical ideas.

DOMAIN VI-MATHEMATICAL LEARNING, INSTRUCTION, AND ASSESSMENT**Competency 020**

The teacher understands how children learn mathematics and plans, organizes, and implements instruction using knowledge of students, subject matter, and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

The beginning teacher:

- A. Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.
- B. Understands how students differ in their approaches to learning mathematics.

- C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- D. Understands how learning may be enhanced through the use of manipulatives, technology, and other tools (e.g., stop watches, scales, rulers).
- E. Understands how to provide instructions along a continuum from concrete to abstract.
- F. Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.
- G. Understands how to create a learning environment that provides all students, including English Language Learners, with opportunities to develop and improve mathematical skills and procedures.
- H. Understands a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.
- I. Understands how to relate mathematics to students' lives and a variety of careers and professions.

Competency 021

The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

The beginning teacher:

- A. Understands the purpose, characteristics, and uses of various assessments in mathematics, including formative and summative assessments.
- B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.
- C. Understands how to develop a variety of assessment and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns.
- D. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor, and modify instruction to improve mathematical learning for all students, including English Language Learner.

Texas Essential Knowledge and Skills for Geometry

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; shapes and figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, algebraic, and coordinate), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities to solve meaningful problems by representing figures, transforming figures, analyzing relationships, and proving things about them.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.

(b) Geometric structure: knowledge and skills and performance descriptions.

(1) The student understands the structure of, and relationships within, an axiomatic system. Following are performance descriptions.

(A) The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems.

(B) Through the historical development of geometric systems, the student recognizes that mathematics is developed for a variety of purposes.

(C) The student compares and contrasts the structures and implications of Euclidean and non-Euclidean geometries.

(2) The student analyzes geometric relationships in order to make and verify conjectures. Following are performance descriptions.

(A) The student uses constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.

(B) The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(3) The student understands the importance of logical reasoning, justification, and proof in mathematics. Following are performance descriptions.

(A) The student determines if the converse of a conditional statement is true or false.

(B) The student constructs and justifies statements about geometric figures and their properties.

(C) The student demonstrates what it means to prove mathematically that statements are true.

(D) The student uses inductive reasoning to formulate a conjecture.

(E) The student uses deductive reasoning to prove a statement.

(4) The student uses a variety of representations to describe geometric relationships and solve problems.

Following is a performance description. The student selects an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(c) Geometric patterns: knowledge and skills and performance descriptions.

The student identifies, analyzes, and describes patterns that emerge from two- and three-dimensional geometric figures. Following are performance descriptions.

(1) The student uses numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.

(2) The student uses properties of transformations and their compositions to make connections between mathematics and the real world in applications such as tessellations or fractals.

(3) The student identifies and applies patterns from right triangles to solve problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

(d) Dimensionality and the geometry of location: knowledge and skills and performance descriptions.

(1) The student analyzes the relationship between three-dimensional objects and related two-dimensional representations and uses these representations to solve problems. Following are performance descriptions.

(A) The student describes, and draws cross sections and other slices of three-dimensional objects.

(B) The student uses nets to represent and construct three-dimensional objects.

(C) The student uses top, front, side, and corner views of three-dimensional objects to create accurate and complete representations and solve problems.

(2) The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. Following are performance descriptions.

(A) The student uses one- and two-dimensional coordinate systems to represent points, lines, line segments, and figures.

(B) The student uses slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons.

(C) The student develops and uses formulas including distance and midpoint.

(e) Congruence and the geometry of size: knowledge and skills and performance descriptions.

(1) The student extends measurement concepts to find area, perimeter, and volume in problem situations. Following are performance descriptions.

(A) The student finds areas of regular polygons and composite figures.

(B) The student finds areas of sectors and arc lengths of circles using proportional reasoning.

(C) The student develops, extends, and uses the Pythagorean Theorem.

(D) The student finds surface areas and volumes of prisms, pyramids, spheres, cones, and cylinders in problem situations.

(2) The student analyzes properties and describes relationships in geometric figures. Following are performance descriptions.

(A) Based on explorations and using concrete models, the student formulates and tests conjectures about the properties of parallel and perpendicular lines.

(B) Based on explorations and using concrete models, the student formulates and tests conjectures about the properties and attributes of polygons and their component parts.

(C) Based on explorations and using concrete models, the student formulates and tests conjectures about the properties and attributes of circles and the lines that intersect them.

(D) The student analyzes the characteristics of three-dimensional figures and their component parts.

(3) The student applies the concept of congruence to justify properties of figures and solve problems. Following are performance descriptions.

(A) The student uses congruence transformations to make conjectures and justify properties of geometric figures.

(B) The student justifies and applies triangle congruence relationships.

(f) Similarity and the geometry of shape: knowledge and skills and performance descriptions. The student applies the concepts of similarity to justify properties of figures and solve problems. Following are performance descriptions.

(1) The student uses similarity properties and transformations to explore and justify conjectures about geometric figures.

(2) The student uses ratios to solve problems involving similar figures.

(3) In a variety of ways, the student develops, applies, and justifies triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples.

(4) The student describes the effect on perimeter, area, and volume when length, width, or height of a three-dimensional solid is changed and applies this idea in solving problems.