Using Multiple Representations to Enhance the Instruction of Sequences and Series
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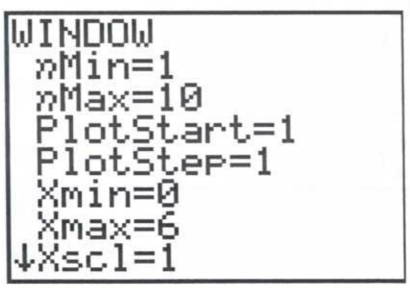
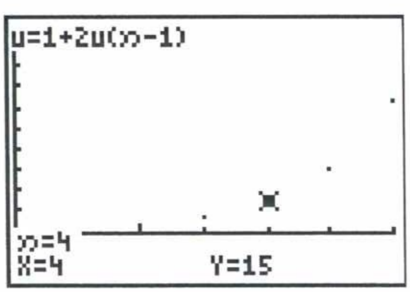
The Basics

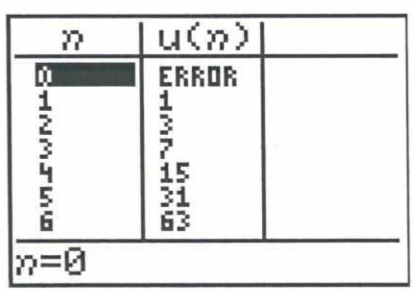
The sequence graphing mode on the TI-84+ graphing calculator allows for the exploration of the graphical representation of sequences—a concept often only explored numerically and symbolically in the past. Both explicit and recursively defined sequences can be graphed on the calculator but the calculator determines values in a sequence term by term regardless of how the sequence is entered. Also regardless of the mode that is used the first term of the sequence MUST be defined in the Y= editor. Just as with other graphical modes on the calculator the sequence can also be explored in a tabular method as well.

Examples:

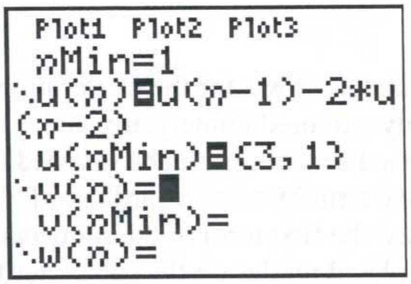
A recursively defined sequence $u_n = 1 + 2u_{n-1}$, $u_1 = 1$

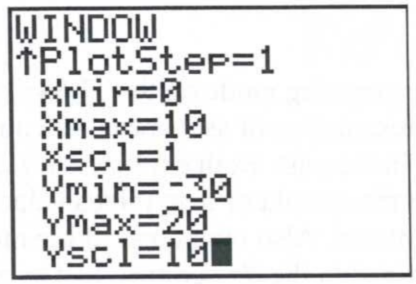


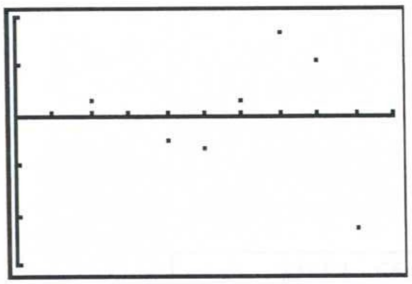




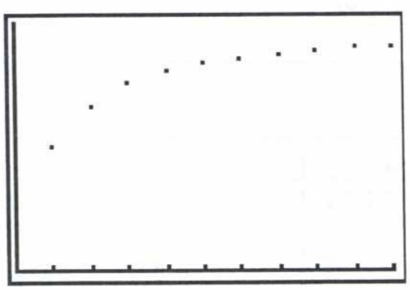
A recursively defined sequence: $u_n = u_{n-1} - 2u_{n-2}$, $u_1 = 1$, $u_2 = 3$







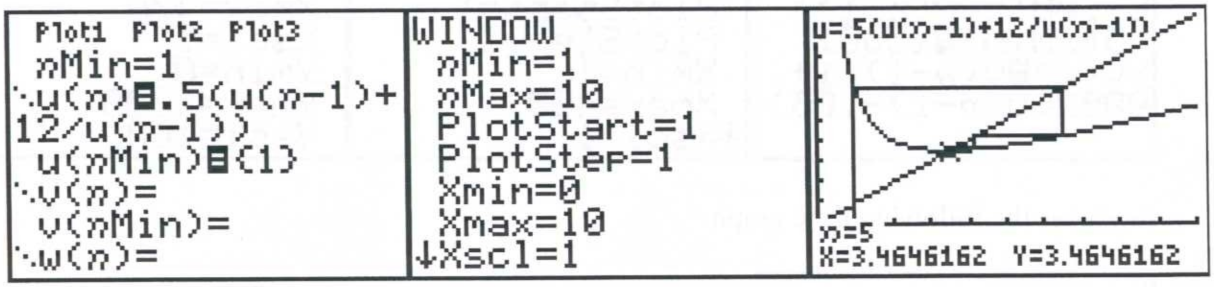
An explicitly defined sequence: $u_n = \dfrac{n}{n+1}$



Web Mode

The web mode of graphing sequences allows you to study the convergence or divergence of recursive sequences. The calculator graphs u_n on the vertical axis and u_{n-1} on the horizontal axis.

In the following example, Newton's Method is restated to allow a view of the convergence to the square root of 12. The square root of any number c can be approximated with this method by allowing $u_n = .5(u_{n-1} + \frac{c}{u_{n-1}})$ and u_{nMin} to be the initial guess for the root.



For further investigation, change the value of u_{nMin} and notice the number of iterations that must be performed before convergance.

Predator-Prey Models

A great application of the sequence mode of the calculator is to study predator-prey models.

Suppose that we have an ecosystem with foxes and rabbits. Let's assume the following initial conditions:

Initial number of rabbits: 500
Initial number of foxes: 50
Rabbit growth rate without foxes per month: .06
Rabbit death rate per foxes per month: .0015
Fox population growth rate per rabbit per month: .0002
Fox population death rate without rabbits per month: .03

Given these conditions then the number of rabbits present in month n is a function of the number of rabbits that were present the previous month, the rabbit population growth rate, the number of foxes and the rabbit population death rate.
In this case:

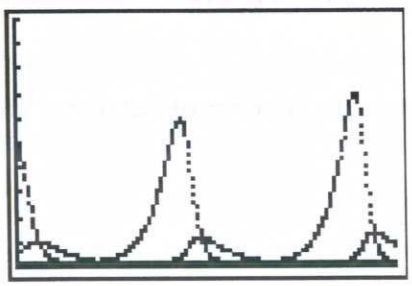
$$R_n = R_{n-1}(1 + .06 - .0015 * F_{n-1})\,;\; R_1 = 500$$

Likewise the fox population is a function of the number of foxes that were present the previous month, the fox population growth rate, the number of rabbits and the fox population death rate:

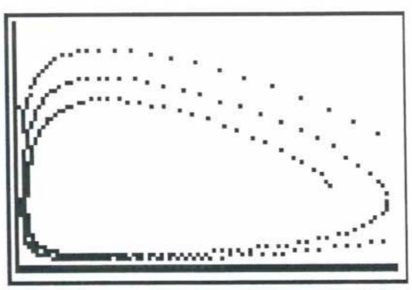
$F_n = F_{n-1}(1 + .0002 * R_{n-1} - .03)$; $F_1 = 50$

Entering this in the calculator:

Produces the following time graph:



But another interesting way to look at this information is by seeing the graph to "*uv*" mode with a window of [0, 600] x [0, 150]:



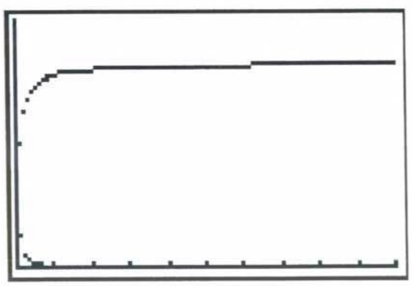
Changing the initial parameters can have interesting effects on the two graphs.

Series

Series can be explored using sequence mode by examining the sequence of partial sums. Since a series must refer to a prior term an adjustment must be made to the sequence rule that is used.

For example to explore the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Producing the following graph:



The draw command can then be used to draw the line $y = \frac{\pi^2}{6}$ to see that the value to which the series converges.