

# Math 102 Fall 2008 Final Exam

Instructor: S. Cautis

Friday, December 12, 2008

*Instructions:* This is a closed book, closed notes exam. Use of calculators is not permitted. You have **3 hours**. Do all 11 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work or explanation will receive little to no credit.

Please print your name clearly here.

Print name: \_\_\_\_\_

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

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Grader's use only:

1. \_\_\_\_\_ /10

2. \_\_\_\_\_ /10

3. \_\_\_\_\_ /20

4. \_\_\_\_\_ /15

5. \_\_\_\_\_ /10

6. \_\_\_\_\_ /10

7. \_\_\_\_\_ /15

8. \_\_\_\_\_ /20

9. \_\_\_\_\_ /20

10. \_\_\_\_\_ /10

11. \_\_\_\_\_ /10

1. [10 points] Evaluate the integral  $\int (\sin x)(\cos x)^2 dx$ .

2. [10 points] Determine whether or not the following improper integral converges. If it converges find its value. If it diverges to  $\pm\infty$  specify which one.

$$\int_{-2}^2 \frac{1}{\sqrt[3]{1+x}} dx$$

3. [20 points] Evaluate the integral  $\int \frac{-x^2+x+9}{x^3+9x} dx$ .

4. [15 points] Evaluate the integral  $\int \frac{1}{1-\sin x} dx$ .

5. [10 points] Does the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{5^{n+1}}$  converge or diverge? If it converges, find its sum.

6. [10 points] Using the power series for  $e^x$  find the power series presentation of  $(x - 1)e^{x^2}$ .

7. [15 points] Find the interval of convergence for the Taylor series  $\sum_{n=1}^{\infty} \frac{(x-2)^n}{3n-1}$ .  
Don't forget to check endpoints.

8. [20 points] Determine whether the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$  converges. If it does determine how many terms you need to add up (how many are sufficient) so that the error is at most  $10^{-5}$ . Make sure you justify your answer.

9. [20 points] Consider the equations  $r = 2 \cos \theta$  and  $\theta = \frac{\pi}{3}$  in polar coordinates.
- (a) Sketch the graphs of both curves on the same plot.
  - (b) Find the points where the two curves intersect and indicate them on the graph in part (a).
  - (c) Find the area of the smaller finite region bounded by the curves.

10. [10 points] Consider the parametric curve  $x = t^3 - 3t$ ,  $y = \sin t$ . Find all points on the curve where the tangent line is vertical.

11. [10 points] Find the arclength of the parametric curve  $x = t^2 - 1$ ,  $y = t^3 + 2$  for  $0 \leq t \leq 2$ .