Sample Test for Second Midterm Exam
Math 102
Spring 2008

Note: The actual exam will be shorter than this sample test.

1. Find the following limits of infinite sequences:

   (a) $\lim_{n\to\infty} \frac{(\ln n)^2}{n}$

   (b) $\lim_{n\to\infty} \sqrt[n]{2^{n+1}}$

2. Find the Taylor polynomial of degree 4 for $f(x) = \cos x$ at $a = \pi$.

3. Does the series $\sum_{n=1}^{\infty} 5\frac{3^n}{4^{n+1}}$ converge or diverge? If it converges, find its sum.

4. For each of the following series decide if it converges or diverges. State the tests which you use.

   (a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

   (b) $\sum_{n=1}^{\infty} \frac{n+2^n}{n+3^n}$

   (c) $\sum_{n=1}^{\infty} \frac{3^n\sqrt{n}}{n!}$

   (d) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$

5. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$.

   (a) Does the series converge?

   (b) Does the series converge absolutely?

6. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n2^n}$.

7. Use the Taylor series for e^x and $\cos x$ to find power series representations of

   (a) $x^2 \cos(x^2)$

   (b) $\frac{e^x - 1}{x}$

Answers to sample test for 2nd midterm

1. (a) 0, (b) 2

2. $P_4(x) = -1 + \frac{1}{2}(x-\pi)^2 + \frac{1}{24}(x-\pi)^4$

3. Converges (geometric series with $r = 3/4$), the sum is $15/4$.

4. (a) Converges by the comparison test, compare with $b_n = 1/n^{4/3}$.

   (b) Converges by the limit comparison test, compare with $b_n = 2^n/3^n$.

   (c) Converges by the ratio test.

   (d) Diverges by the integral test, evaluate the improper integral of $\frac{1}{x(\ln x)^2}$ by the substitution $u = \ln x$.

5. (a) Converges by the alternating series test.

   (b) Does not converge absolutely: Use Limit Comparison Test with $b_n = 1/n$ to show divergence of $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$.

6. $[-1, 3)$

7. (a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n)!} = x^2 - \frac{x^6}{2!} + \frac{x^{10}}{4!} - \frac{x^{14}}{6!} + \ldots$

   (b) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n!} = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \ldots$