

Math 102 Spring 2008: Solutions: HW #2

Instructor: S. Cautis

1. section 7.4, #2 Use the identity $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$ to get,

$$\begin{aligned}\int \cos^2(5x) dx &= \frac{1}{2} \int 1 + \cos(10x) dx \\ &= \frac{1}{2} \left(x + \frac{1}{10} \sin(10x) \right) + C \\ &= \frac{1}{2} x + \frac{1}{20} \sin(10x) + C\end{aligned}$$

2. section 7.4, #10 Use that $\cot x = \frac{\cos x}{\sin x}$ and the identity $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$

$$\begin{aligned}\int \sin^2 x \cot^2 x dx &= \int \sin^2 x \frac{\cos^2 x}{\sin^2 x} dx \\ &= \int \cos^2 x dx \\ &= \frac{1}{2} \int 1 + \cos(2x) dx \\ &= \frac{1}{2} \left(x + \frac{1}{2} \sin(2x) \right) + C \\ &= \frac{1}{2} x + \frac{1}{4} \sin(2x) + C\end{aligned}$$

3. section 7.4, #24 Use that $\tan^2 x = \sec^2 x - 1$

$$\begin{aligned}\int \tan^3 x dx &= \int (\sec^2 x - 1) \tan x dx \\ &= \int \sec^2 x \tan x dx - \int \tan x dx \\ &= \frac{\tan^2 x}{2} + \ln |\cos x| + C\end{aligned}$$

4. section 7.4, #26

Use the identity $\sec^2 \theta = \tan^2 \theta + 1$ and then set $u = \tan \theta$, then $du = \sec^2 \theta d\theta$ we have:

$$\begin{aligned}
\int \tan \theta \sec^4 \theta d\theta &= \int \tan \theta (\tan^2 \theta + 1) \sec^2 \theta d\theta \\
&= \int u(u^2 + 1) du \\
&= \int u^3 + u du \\
&= \frac{1}{4} u^4 + \frac{1}{2} u^2 + C \\
&= \frac{1}{4} \tan^4 \theta + \frac{1}{2} \tan^2 \theta + C
\end{aligned}$$

5. section 7.4, #34

Use the identities $\sec \theta = \frac{1}{\cos \theta}$ and $\sec^2 \theta = \tan^2 \theta + 1$ and then set $u = \tan 2x$, then $du = 2 \sec^2 2x dx$. So we find

$$\begin{aligned}
\int \frac{1}{\cos^4 2x} dx &= \int \sec^4 2x du \\
&= \int (\tan^2 2x + 1) \sec^2 2x dx \\
&= \frac{1}{2} \int (u^2 + 1) du \\
&= \frac{1}{6} u^3 + \frac{1}{2} u + C \\
&= \frac{1}{6} \tan^3 2x + \frac{1}{2} \tan 2x + C
\end{aligned}$$

6. section 7.4, #48

To find the area between two curves we graph them and see which is above the other on the interval over which we are integrating, here $\sin^3 x$ is above $\cos^3 x$, so the integral is:

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin^3 x - \cos^3 x dx$$

Also the area is symmetric about $x = \frac{3\pi}{4}$ so this is the integral:

$$2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^3 x - \cos^3 x dx$$

Now we break it into two different integrals use the identity $\cos^2 + \sin^2 x = 1$ and do substitution on both of them.

$$2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^3 x - \cos^3 x dx = 2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1 - \cos^2 x) \sin x dx - 2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1 - \sin^2 x) \cos x dx$$

Let $u = \cos x$ so $du = -\sin x dx$ for the first integral, and let $v = \sin x$ so $du = \cos x dx$ for the second. Now $\cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ then we have:

$$\begin{aligned} &= 2 \int_{\frac{\sqrt{2}}{2}}^{-\frac{\sqrt{2}}{2}} 1 - u^2 (-du) - 2 \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} 1 - v^2 dv \\ &= -2(u - \frac{u^3}{3}) \Big|_{\frac{\sqrt{2}}{2}}^{-\frac{\sqrt{2}}{2}} - 0 \\ &= -2(-\frac{\sqrt{2}}{2} - \frac{(-\frac{\sqrt{2}}{2})^3}{3} - [\frac{\sqrt{2}}{2} - \frac{(\frac{\sqrt{2}}{2})^3}{3}]) \\ &= -2(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{12} - [\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{12}]) \\ &= 2\sqrt{2} - \frac{\sqrt{2}}{3} = \frac{5\sqrt{2}}{3}. \end{aligned}$$

7. section 7.4, #60

Use the identity $\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$ so we have,

$$\begin{aligned} \int \frac{1}{2}(\cos(2x) - \cos(6x)) dx &= \frac{1}{2}(\frac{1}{2} \sin(2x) - \frac{1}{6} \sin(6x)) + C \\ &= \frac{1}{4} \sin(2x) - \frac{1}{12} \sin(6x) + C \end{aligned}$$

8. section 7.5, #2

The degree of x^3 is greater than the degree of $2x - 1$ so we must first divide, to get,

$$\frac{x^3}{2x - 1} = \frac{1}{2}x^2 + \frac{1}{4}x + \frac{1}{8} + \frac{1}{8} \frac{1}{2x - 1}.$$

Then

$$\int \frac{1}{2}x^2 + \frac{1}{4}x + \frac{1}{8} + \frac{1}{8} \frac{1}{2x - 1} dx = \frac{1}{6}x^3 + \frac{1}{8}x^2 + \frac{1}{8}x + \frac{1}{8} \ln |2x - 1| + C$$

9. section 7.5, #6

The degree of x^3 is greater than the degree of $x^2 + x - 6$ so we must first divide, to get,

$$\frac{x^3}{x^2 + x - 6} = x - 1 + \frac{7x - 6}{x^2 + x - 6}.$$

Now

$$\frac{7x - 6}{x^2 + x - 6} = \frac{7x - 6}{(x + 3)(x - 2)} = \frac{A}{x + 3} + \frac{B}{x - 2}.$$

Multiply to get the same denominator and set the coefficients of the numerators equal to get:

$$\begin{aligned} 7 &= A + B \\ -6 &= -2A + 3B \end{aligned}$$

Solving the equations gives $A = \frac{27}{5}$ and $B = \frac{8}{5}$.

$$\int x - 1 + \frac{27}{5} \frac{1}{x + 3} + \frac{8}{5} \frac{1}{x - 2} dx = \frac{1}{2}x^2 + x + \frac{27}{5} \ln|x + 3| + \frac{8}{5} \ln|x - 2| + C$$

10. section 7.5, #12

The degree of $2x^3 - 1$ is greater than the degree of $x^2 + 1$ so we must first divide, to get,

$$\frac{2x^3 - 1}{x^2 + 1} = 2x + \frac{-2x - 1}{x^2 + 1}.$$

In $\frac{-2x-1}{x^2+1}$ the denominator is an irreducible quadratic.