Math 102 Spring 2008: **Solutions: HW #4**

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1. section 10.2, #6

   The denominators are always one more than a square. So the nth denominator is $n^2 + 1$ so the general term in the sequence is

   $$a_n = \frac{1}{n^2+1}.$$

2. section 10.2, #12

   Dividing top and bottom by n^2 we get $\frac{n}{10+1/n}$. Now $10 + 1/n \to 10$ as $n \to \infty$ so we get that $\frac{n}{10+1/n}$ must diverge.

3. section 10.2, #14

   We have $(-1/2)^n = (-1)^n \frac{1}{2^n} \to 0$ as $n \to \infty$. Thus

   $$\lim_{n\to\infty} 2 - (-1/2)^n = 2.$$

4. section 10.2, #20

   We have $-1 \leq \cos n \leq 1$ so $1 \leq 2 + \cos n \leq 3$ and hence

   $$\frac{1}{n} \leq \frac{2+\cos n}{n} \leq \frac{3}{n}$$

   Thus

   $$\frac{1}{\sqrt{n}} \leq \sqrt{\frac{2+\cos n}{n}} \leq \frac{3}{\sqrt{n}}$$

   Since $\frac{1}{\sqrt{n}} \to 0$ this means that

   $$\sqrt{\frac{2+\cos n}{n}} \to 0$$

   as $n \to \infty$.

5. section 10.2, #30

   Using L'hôpital's rule four times we have

   $$\begin{aligned} \lim_{x\to\infty} \frac{x^3}{e^{x/10}} &= \lim_{x\to\infty} \frac{3x^2}{e^{x/10}/10} \\ &= \lim_{x\to\infty} \frac{6x}{e^{x/10}/100} \\ &= \lim_{x\to\infty} \frac{6}{e^{x/10}/1000} \\ &= \lim_{x\to\infty} 0 \end{aligned}$$

   This means that the sequence converges and the limit is zero.

6. section 10.2, #34

   Let's look at $b_n = \ln a_n = \frac{1}{n}\ln(2n+5)$. Now by L'hôpital's rule

$$\lim_{x\to\infty} \frac{\ln(2x+5)}{x} = \lim_{x\to\infty} \frac{2/(2x+5)}{1} = 0$$

   So $b_n \to 0$ which means $a_n \to 1$.

7. section 10.2, #50

   We have

$$\lim_{n\to\infty} \frac{3n-1}{4n+1} = \lim_{n\to\infty} \frac{3-1/n}{4+1/n} = \frac{3}{4}$$

   where we divided the top and bottom by n to get the first equality. Thus

$$\lim_{n\to\infty} 3\sin^{-1}\sqrt{\frac{3n-1}{4n+1}} = 3\sin^{-1}(\lim_{n\to\infty}\sqrt{\frac{3n-1}{4n+1}}) = 3\sin^{-1}\sqrt{\frac{3}{4}} = \pi.$$

8. section 10.2, #56

   Since $a_{n+1} = 1 + (1/a_n)$ and the limit $\lim_{n\to\infty} a_n = L$ exists we can take the limit of both sides to get

$$L = 1 + 1/L.$$

   This gives $L^2 - L - 1 = 0$ which means (by the quadratic formula) that

$$L = \frac{1 \pm \sqrt{5}}{2}$$

   Since L is clearly non-negative this means

$$L = \frac{1+\sqrt{5}}{2}.$$