Math 102 Spring 2008: **Solutions: HW #8**

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section 10.5, #36 If $p \leq 0$ then $-p \geq 0$ and $\sum_{n=1}^{\infty} n(n^2+1)^{-p}$ diverges, so consider $p > 0$. If $p \neq 1$

$$\int_1^{\infty} \frac{x}{(x^2+1)^p} dx = \left[\frac{1}{2(1-p)(x^2+1)^{p-1}} \right]_1^{\infty},$$

which diverges for $0 < p < 1$ and converges for $p > 1$.

Next, if $p = 1$ then

$$\int_1^{\infty} \frac{x}{(x^2+1)^p} dx = \left[\frac{1}{2} \ln(x^2+1) \right]_1^{\infty} = +\infty$$

So the sum diverges for $p \leq 1$ and converges for $p > 1$.

section 10.5, #40 We need to find n such that $R_n < 0.00005$,

$$R_n \leq \int_n^{\infty} \frac{1}{x^2} dx = \lim_{a \to \infty} \left(\frac{1}{n} - \frac{1}{a} \right) = \frac{1}{n}.$$

So $\frac{1}{n} < 0.00005$, therefore $n > 20000$.

section 10.5, #46 We need to find n such that $R_n < 5 \times 10^{-8} = 0.00000005$,

$$R_n \leq \int_n^{\infty} \frac{1}{x^7} dx = \lim_{a \to \infty} \left(\frac{1}{6n^6} - \frac{1}{6a^6} \right) = \frac{1}{6n^6}.$$

So $\frac{1}{6n^6} < 0.00000005$, therefore $n > 12.2221$. So take $N = 13$ and find S_N using some program like maple or mathematica and rounding appropriately we have,

$$S_N = \sum_{n=1}^{13} \frac{1}{x^7} \approx 1.008349250.$$

section 10.6, #2 By using the limit comparison test, and comparing with the harmonic series, we see

$$\lim_{n \to \infty} \frac{\frac{n^3+1}{n^4+2}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n^4+n}{n^4+2} = 1 > 0.$$

Thus the series diverges since the harmonic series diverges.

section 10.6, #10 Using the limit comparison test, and comparing with the harmonic series, we see

$$\lim_{n \to \infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n}{\sqrt{n^2+1}} = 1 > 0.$$

Thus the series diverges since the harmonic series diverges.

section 10.6, #16 The series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$ is dominated by $\sum_{n=1}^{\infty} \frac{1}{3^n}$. The series $\sum_{n=1}^{\infty} \frac{1}{3^n}$ is a geometric series with $r = \frac{1}{3} < 1$ so it converges. Thus by the comparison test $\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$ converges.

section 10.6, #26 By using the limit comparison test, and comparing with the harmonic series, we see

$$\lim_{n\to\infty} \frac{\frac{n^2+2}{n^3+3n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3+2n}{n^3+3n} = 1 > 0.$$

Thus the series diverges since the harmonic series diverges.

section 10.6, #34 The series $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ is dominated by $\sum_{n=1}^{\infty} \frac{n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a p-series with $p > 1$ so it converges. Thus by the comparison test $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ converges.

section 10.6, #38 If we take the first ten terms in the series we get,

$$S_{10} = \sum_{n=1}^{10} \frac{1}{3^n+1} = \frac{76043801855199427217}{190429124708983981100} \approx 0.404054799773.$$

Since $\frac{1}{3^n+1} \leq \frac{1}{3^n}$, we can estimate the error as follows:

$$R_{10} \leq \sum_{n=11}^{\infty} \frac{1}{3^n} = \sum_{m=0}^{\infty} \frac{1}{3^{m+11}} = \frac{2}{3^{10}}.$$

section 10.6, #48 Since the sequence $\{c_n\}$ converges to zero, there exists a positive integer Nsuch that $0 \leq c_n a_n \leq a_n$ for $n \geq N$. So $\sum_{n=N}^{\infty} c_n a_n$ is dominated by $\sum_{n=N}^{\infty} a_n$, thus by the comparison test $\sum_{n=N}^{\infty} a_n$ converges. Therefore $\sum c_n a_n = S_{N-1} + \sum_{n=N}^{\infty} c_n a_n$ converges.