

Math 102 Spring 2008: Solutions: HW #9

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1. section 10.7, #4

The sequence $\{\frac{n}{3n^2+2}\}$ is monotonically decreasing with limit 0, so the given series meets both criteria of the alternating series test.

Therefore this series converges.

2. section 10.7, #12

The series diverges by the nth-term test for divergence because $(\frac{n\pi}{10})^{n+1}$ goes to $+\infty$ as n goes to $+\infty$.

3. section 10.7, #26

The ratio test yields $\rho = \lim_{n \rightarrow \infty} \frac{3^{n+1}n!n}{3^n(n+1)!(n+1)} = \lim_{n \rightarrow \infty} \frac{3n}{(n+1)^2} = 0$.

Therefore the given series converges absolutely.

4. section 10.7, #32

Because $\lim_{n \rightarrow \infty} \frac{2^{3n}}{7^n} = \lim_{n \rightarrow \infty} \frac{8^n}{7^n} = \lim_{n \rightarrow \infty} (\frac{8}{7})^n = +\infty$, the series diverges by the nth-term test for divergence.

5. section 10.7, #40

The ratio test yields

$$\rho = \lim_{n \rightarrow \infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \cdot (2n+1) \cdot 1 \cdot 4 \cdot 7 \cdots (3n-2)}{1 \cdot 3 \cdot 5 \cdots (2n-1) \cdot 1 \cdot 4 \cdot 7 \cdots (3n-2) \cdot (3n+1)} = \lim_{n \rightarrow \infty} \frac{2n+1}{3n+1} = \frac{2}{3}.$$

Because $\rho < 1$, the original series converges absolutely.

6. section 10.7, #52

The condition $\frac{1}{(2n)!} < 0.000005$ leads to $4 < n < 5$, so the sum of the terms through $n = 4$ will provide five-places accuracy. The sum of the first terms of the series is

$$\sum_{n=0}^4 \frac{(-1)^n}{(2n)!} = \frac{4357}{8064} \cong 0.5403025$$

so to five places, the sum of the infinite series is 0.54030.

7. section 10.7, #60

Part(a), The ratio test yields

$$\rho = \lim_{n \rightarrow \infty} \frac{(n+1)|r|^{n+1}}{n|r|^n} = \lim_{n \rightarrow \infty} \frac{(n+1)|r|}{n} = |r| < 1.$$

Therefore the series in question converges.

Part(b), Let S denote the sum of the series in Part(a). Then

$$\begin{aligned}
 (1-r)S &= \lim_{k \rightarrow \infty} \sum_{n=0}^k (nr^n - nr^{n+1}) \\
 &= \lim_{k \rightarrow \infty} (r - r^2 + 2r^2 - 2r^3 + 3r^3 - 3r^4 + \dots + (k-1)r^{k-1} - (k-1)r^k + kr^k - kr^{k+1}) \\
 &= \lim_{k \rightarrow \infty} (1 + r + r^2 + r^3 + \dots + r^k - kr^{k+1} - 1) \\
 &= \lim_{k \rightarrow \infty} \left(\frac{1-r^{k+1}}{1-r} - kr^{k+1} - 1 \right) \\
 &= \frac{r}{1-r}.
 \end{aligned}$$

Thus $\sum_{n=0}^{\infty} nr^n = S = \frac{r}{(1-r)^2}$.

8. section 10.8, #8

The ratio test yields

$$\lim_{n \rightarrow \infty} \frac{(2n+1)^{1/2} 4^{n+1} |x|^{n+1}}{(2n+3)^{1/2} 4^n |x|^n} = 4|x|.$$

So the series converges if $-\frac{1}{4} < x < \frac{1}{4}$. When $x = 1/4$, the series converges by the alternating series test. When $x = -1/4$, the series diverges by limit-comparison test with the p-series for which $p = 1/2$, hence $(-1/4, 1/4]$.

9. section 10.8, #26

The ratio test yields

$$\lim_{n \rightarrow \infty} \frac{n \cdot 10^n \cdot |x-2|^{n+1}}{(n+1) \cdot 10^{n+1} \cdot |x-2|^n} = \frac{|x-2|}{10},$$

and therefore this series converges if $-10 < x-2 < 10$, that is, if $-8 < x < 12$. If $x = 12$ it converges by the alternating series test; if $x = -8$ it diverges because it becomes harmonic series, hence $(-8, 12]$.

10. section 10.8, #28

Notice that the given series is actually geometric with first term 1 and ratio $r = \frac{x^2+1}{5}$. Hence it converges if $-1 < r < 1$, that is, $-2 < x < 2$. So its interval of convergence is $(-2, 2)$ and its sum is $\frac{5}{4-x^2}$.