Lecture 12: Review Chapter 7

2/7/06

Note Title 2/7/2006

In Chapter 7, we covered

I. Techniques of Integration

A. Substitution

- simple substitution

- $\int f(g(x))\,g'(x) = \int f(u)\,du$ (letting $u = g(x)$)

Ex. $\int 6x^5 (x^6+3)^4\,dx$ let $u = x^6+3$ $du = 6x^5$

$= \int u^4\,du$

- trig substitution (special form of simple subst)

if integral contains

$a^2 - u^2 \longrightarrow$ let $u = a\sin\theta$

$a^2 + u^2 \longrightarrow$ let $u = a\tan\theta$

$u^2 - a^2 \longrightarrow$ let $u = a\sec\theta$

} especially useful for integrals w/ $\sqrt{a^2-u^2}$, etc.

Ex. $\int \sqrt{x^2-9}\,dx$ $\quad x = 3\sec\theta$, $dx = 3\sec\theta\tan\theta\,d\theta$

$= \int 3\tan\theta \cdot 3\sec\theta\tan\theta\,d\theta$

B. Integration by parts

$\int u\,dv = uv - \int v\,du$

Useful if $\int f(x)\,g(x)\,dx$ where f is easy to diff.
g is easy to integrate

Ex. $\int x\sin(x)\,dx$ $\quad u = x \quad dv = \sin(x)\,dx$

$du = dx \quad v = -\cos(x)$

$= -x\cos(x) + \int \cos(x)\,dx$

C. Other special methods

- partial fraction decomposition

- $\frac{p(x)}{q(x)} \xrightarrow{\text{long division}} r(x) + \frac{s(t)}{q(t)}$ where degree $s(t)$ < deg $q(t)$

- $\frac{s(t)}{r(t)}$ = sum of "easier" integral like $\frac{1}{x+1}, \frac{1}{x^2+1}$, etc.

Ex: $\frac{x}{x^2-5x+6} = \frac{x}{(x-3)(x-2)} = \frac{A}{x-2} + \frac{B}{x-3}$

Solve $A = -2$, $B = 3 \Rightarrow$

$$\int \frac{x}{x^2-5x+6}\,dx = \int \frac{-2}{x-2}\,dx + \int \frac{3}{x-3}\,dx$$

- products of trig functions

$\int \sin^m(x)\cos^n(x)\,dx$:

Case 1: n odd | use $u = \sin(x)$, pull out $\cos(x)$ for du, use $\cos^2(x) = 1 - \sin^2(x)$ for other cosines

Case 2: m odd | use $u = \cos(x)$ pull out $-\sin(x)$ for du use $\sin^2(x) = 1 - \cos^2(x)$ for other sines

Case 3: n, m even | Use ½-angle identities

$$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$$

[similar methods for $\tan^m(x)\sec^n(x)$ and $\cot^m(x)\csc^n(x)$]

Should know the integrals of ...

- $\cos(x), \sin(x), e^x, \sec^2(x), \csc^2(x), \sec(x)\tan(x), \csc(x)\cot(x), 1/x$, etc.

- Completing the square

If integral contains $\sqrt{ax^2+bx+c}$ or an "ax^2+bx+c" in the denominator

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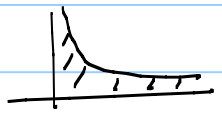
complete the square to get $\pm(u^2 \pm c^2)$ and then use trig substitution

Ex: $\int \sqrt{x^2+4x+1}\,dx = \int \sqrt{(x+2)^2-3}\,dx$

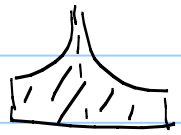
let $x+2 = \sqrt{3}\sec\theta$ (trig. substitution)

II. Improper Integral

A. $\int_a^\infty f(x)\,dx$ or $\int_{-\infty}^a f(x)\,dx$ or $\int_{-\infty}^\infty f(x)\,dx$



B. $\int_a^b f(x)\,dx$ when f not defined at $x=c$



Ex: $\int_0^\infty \frac{1}{x^3}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{x^3}\,dx$ ...