18.904 HOMEWORK THREE
DUE 3-1-2005

Complete the following problems. You are encouraged to work with other students in the class on the problems. Please write up your own solutions. Each problem is worth 10 points.

(1) Let $\{U_i\}$ be an open covering of the space X having the following properties:
   (a) There exists a point x_0 such that $x_0 \in U_i$ for all i.
   (b) Each U_i is simply connected.
   (c) If $i \neq j$ then $U_i \cap U_j$ is path-connected.

   Prove that X is simply connected.

   [Hint: To prove any path $f : I \to X$ based at x_0 is trivial, consider the open covering $\{f^{-1}(U_i)\}$ of I and use the fact that I is compact. If you wish, you may use the fact that I is a compact metric space hence every open covering of I has a Lebesgue number ϵ (see for example, Lemma 7.2 of Munkres "Topology" for the proof). Recall that ϵ is a *Lebesgue number* of a covering of a metric space Y if the following condition holds: Any subset of Y of diameter $< \epsilon$ is contained in some set of the covering.]

(2) Use (1) to show that S^n is simply connected if $n \geq 2$.