# 18.905 HOMEWORK EIGHT
# DUE 11-3-2004 BY 3PM

Reading Assignment for the week:

pp. 197–204 Cohomology of Spaces

Complete the following problems. You are encouraged to work with other students in the class on the problems. You may also consult Hatcher's book on Algebraic Topology. However, you must write up your own solutions. Please use complete sentences when writing up your solutions.

1. Let $X$ be the 2-dimensional CW-complex obtained by attaching a disk $D$ to a wedge of two circles where the attaching map is $xy^2x^2y^2x^{-1}y^{-1}$ (here the circles are oriented and are labelled by $x$ and $y$). Note that this space can also be described by the identification space of a 9 sided polygon with sides labelled $xyyxxyyx^{-1}y^{-1}$. Using cellular homology, compute the homology of $X$.

2. Use cellular homology to compute the homology of the 5-skeleton of the 10-dimensional simplex, $\Delta^{10}$. Hint: Let $X$ be the 10-dimensional simplex. Compare the cellular chain complex $X^5$ with the cellular chain complex of $X$.

3. Use cellular cohomology to compute the cohomology of $S^1 \times S^1$ with coefficients both in $\mathbb{Z}$ and $\mathbb{Z}_2$. Do the same for the Klein bottle.

4. Exercise 4, p. 205

5. Show that the contravariant functor $\mathrm{Hom}(-, G)$ is right exact. That is, show that if
$$A \to B \to C \to 0$$
is an exact sequence then
$$\mathrm{Hom}(A, G) \leftarrow \mathrm{Hom}(B, G) \leftarrow \mathrm{Hom}(C, G) \leftarrow 0$$
is exact.

† Late homeworks will not be graded and will receive at most 50% of the total grade.