18.905 HOMEWORK ELEVEN
DUE MONDAY, 11-29-2004 BY 3PM

Reading Assignment for the week:

Mon 11-22: pp. 233-239, Poincare Duality (Orientations and Homology)
Wed 11-24: no class (Have a Happy Turkey Day!)

Complete the following problems. You are encouraged to work with other students in the class on the problems. You may also consult Hatcher's book on Algebraic Topology. However, you must write up your own solutions. Please use complete sentences when writing up your solutions.

1. Let C_n be the singular n-chain functor from the category of topological spaces and continuous maps to the category of abelian groups and homomorphisms. Show that there is a natural transformation T from C_m to C_n when $m \leq n$. See p. 165 for the definition of a natural transformation.

2. You showed in a previous assignment that the cellular chain complexes for $\mathbb{C}\mathrm{P}^2$ and $S^2 \vee S^4$ are isomorphic but that these two spaces are not homeomorphic. Compute the cohomology ring of $S^2 \vee S^4$ (with coefficients in $\mathbb{Z}$) without using the result from Example 3.13. Using the cohomology rings of these two spaces, show that $\mathbb{C}\mathrm{P}^2$ is not homotopy equivalent to $S^2 \vee S^4$.

† Late homeworks will not be graded and will receive at most 50% of the total grade.