

**18.905 HOMEWORK NINE**  
**DUE 11-10-2004 BY 3PM**

Reading Assignment for the week:

Mon 11-08: pp. 261–267, Universal Coefficients for Homology

Wed 11-10: pp. 206–211, Cup Product

Complete the following problems. You are encouraged to work with other students in the class on the problems. You may also consult Hatcher's book on Algebraic Topology. However, you must write up your own solutions. Please use complete sentences when writing up your solutions.

1. Exercise 1, p. 204
2. Exercise 9, p. 205
3. Let  $X$  be any topological space with finitely generated homology groups.
  - (a) Give an explicit expression (with proof) for the cohomology of  $X$  with coefficients in  $\mathbb{Q}$ ,  $\mathbb{R}$  and  $\mathbb{C}$  in terms of the Betti numbers of  $X$  and  $\mathbb{Q}$ ,  $\mathbb{R}$  and  $\mathbb{C}$  respectively.
  - (b) Give an explicit expression (with proof) for the cohomology of  $X$  with coefficients in  $\mathbb{Z}_2$  in terms of the Betti numbers of  $X$ , (some) of the torsion coefficients of  $X$  and  $\mathbb{Z}_2$  (see p. 130 in Hatcher for the definition of Betti numbers and torsion coefficients).
4. Show that  $S^2 \vee S^4$  and  $\mathbb{C}P^2$  have the identical cellular chain complexes (using the obvious cellular structure). Conclude that they have isomorphic cohomology groups (with any coefficient group  $G$ ). Are they homeomorphic, why or why not?
5. Compute the cohomology groups of  $\mathbb{R}P^n$  and  $\mathbb{R}P^\infty$  with  $\mathbb{Z}_2$  coefficients.

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† Late homeworks will not be graded and will receive at most 50% of the total grade.