18.905 HOMEWORK SEVEN
DUE 10-27-2004 BY 3PM

Reading Assignment for the week:

Mon 10-25: pp. 185–190 Idea of Cohomology
Wed 10-27: pp. 190–197 Universal Coefficient Theorem

Complete the following problems. You are encouraged to work with other students in the class on the problems. You may also consult Hatcher's book on Algebraic Topology. However, you must write up your own solutions. Please use complete sentences when writing up your solutions.

1. A *platonic solid* is a convex polyhedron in $\mathbb{R}^3$ all of whose faces are congruent regular polygons, and where the same number of faces meet at every vertex. The best know example is a cube (or hexahedron) whose faces are six congruent squares. Using Euler characteristic, show that are are exactly 5 platonic solids. These five are familiarly known as the (1) Tetrahedron (4 faces), (2) Octahedron (8 faces), (3) Icosahedron (20 faces), (4) Hexahedron (6 faces), and (5) Dodecahedron (12 faces).
2. (a) Suppose $0 \to A \xrightarrow{\alpha} B \xrightarrow{\beta} C \to 0$ is a short exact sequence of finitely generated abelian groups (i.e. A, B, and C are all finitely generated abelian groups). Prove that $\text{rank } B = \text{rank } A + \text{rank } C$.
   (b) Suppose
   $$0 \to A_n \xrightarrow{\alpha_n} \cdots \xrightarrow{\alpha_{i+2}} A_{i+1} \xrightarrow{\alpha_{i+1}} A_i \xrightarrow{\alpha_i} A_i \xrightarrow{\alpha_{i-1}} \cdots \xrightarrow{\alpha_1} A_0 \to 0$$
   is a long exact sequence of finitely generated abelian groups. Prove that
   $$\sum_{i=0}^{n} (-1)^i \text{ rank } A_i = 0$$
3. Exercise 2, p. 155 (You do NOT have to construct maps from $\mathbb{R}\mathrm{P}^{2n-1}$ to itself without fixed points.)
4. Exercise 8, p. 155 (Only do the first part. You do NOT need to show that the local degree of $\hat{f}$ at a root of f is the multiplicity of the root.)
5. Exercise 15, p. 156 (see the diagram on p. 139)

† Late homeworks will not be graded and will receive at most 50% of the total grade.