

**18.905 HOMEWORK THREE**  
**DUE 9-29-2004 BEFORE 3PM<sup>†</sup>**

Reading Assignment for the week:

Mon 9-27 (was listed as material for Wed 9-22 in homework two):

Chapter 2, 108–113 (Singular Homology and Homotopy Invariance; stop before Exact Sequences)

Wed 9-29:

Chapter 2, 113–118 (Exact Sequences)

Complete the following problems. You are encouraged to work with other students in the class on the problems. You may also consult Hatcher's book on Algebraic Topology. However, you must write up your own solutions. Please use complete sentences when writing up your solutions.

1. There is an obvious  $\Delta$ -complex,  $X$ , for the wedge of  $g$  circles obtained with  $g$  1-simplices. Using this complex, compute  $H_1^\Delta(X)$  and  $H_2^\Delta(X)$ .
2. Exercise 5, p. 131
3. Compute  $H_1^\Delta(X)$  and  $H_2^\Delta(X)$  for some  $\Delta$ -complex,  $X$ , of the quotient space indicated in Figure 1. This space is called the “5-fold dunce cap.” Define analogously the “ $k$ -fold dunce cap” and speculate on its first and second simplicial homology groups. Hint: A  $\Delta$ -complex can be obtained by adding a vertex in the middle and 5 (or  $k$ ) 1-simplices pointing radially inward.

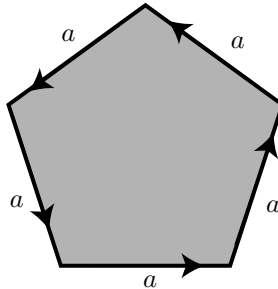


FIGURE 1. The 5-fold dunce cap

4. Given a finitely generated abelian groups  $G_1$  and  $G_2$ , with  $G_2$  free, describe (without proof) a finite 2-dimensional  $\Delta$ -complex (a complex with a finite number of 0, 1, and 2-simplices)  $K$  such that  $|K|$ , the underlying topological space of  $K$ , is connected,  $H_1^\Delta(K) \cong G_1$ , and  $H_2^\Delta(K) \cong G_2$ .

---

<sup>†</sup> Late homeworks will not be graded and will receive at most 50% of the total grade.