18.905 HOMEWORK TWO
DUE 9-22-2004 BEFORE 3PM

Reading Assignment for the week of 9-15 through 9-22:

Wed 9-15 (This means I *plan* to cover this material on Wed 9-15):
    Chapter 1, pp. 40–44 (do not read the proof of Van Kampen's Theorem starting on p. 44)
    Chapter 1, Example 1.23 on pp. 46–47
    Chapter 1, pp. 50–52
Mon 9-20:
    Chapter 2, 97–101 (The idea of Homology)
    Chapter 2, 102–107 (Simplicial Homology)
Wed 9-22:
    Chapter 2, 108–113 (Singular Homology and Homotopy Invariance; stop before Exact Sequences)

Complete the following problems. You are encouraged to work with other students in the class on the problems. You may also consult Hatcher's book on Algebraic Topology. However, you must write up your own solutions. Please use complete sentences when writing up your solutions.

1. Recall that a space that is homotopy equivalent to a point is called *contractible*. Let X be a topological space. Show that the following conditions on X are equivalent.
   (a) X is contractible
   (b) The identity map on X is nullhomotopic (homotopic to a constant map).
   (c) Every map $f : X \to Y$, for arbitrary Y, is nullhomotopic.
   (d) Every map $f : Y \to X$, for arbitrary Y, is nullhomotopic.
2. Exercise 16(c), p. 39
3. For each of the following spaces, the fundamental group is either isomorphic to the trivial group, $\mathbb{Z}$, or $\mathbb{Z} * \mathbb{Z}$. Determine for each space which of the three alternatives holds. You do not need to show your work on this problem.
   (a) The "solid torus," $D^2 \times S^1$
   (b) The torus $S^1 \times S^1$ with a point removed
   (c) The cylinder $S^1 \times I$
   (d) The infinite cylinder $S^1 \times \mathbf{R}$
   (e) $\mathbf{R}^3$ with the nonnegative x, y and z axes removed
   The following subsets of $\mathbf{R}^2$:
   (f) $\{x \mid \|x\| > 1\}$
   (g) $\{x \mid \|x\| \geq 1\}$
   (h) $\{x \mid \|x\| < 1\}$
   (i) $\{x \mid \|x\| = 1\} \cup (\mathbf{R} \times \{0\})$
4. Exercise 7, p. 53
5. Let X be the 1-complex which is the union of the edges (and vertices) on a tetrahedron. Compute $\pi_1(X, x_0)$ where x_0 is a basepoint in X.