Linear Algebra

1. Consider the following sentence:

The vector x is a linear combination of v_1 and v_2.

   Write an equivalent statement that uses the word "span" as a noun.

2. Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$. Find and draw five different vectors in the span of $\mathbf{v_1}$.

   What does the span of $\mathbf{v_1}$ look like in $\mathbb{R}^3$?

   Can a single vector span $\mathbb{R}^3$?

3. (a) Let $\mathbf{v_1}$ be as above and let $\mathbf{v_2} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

   (b) Is $\mathbf{v_1}$ in the span of $\{\mathbf{v_1}; \mathbf{v_2}\}$?

   (c) Is $\mathbf{v_2}$ in the span of $\{\mathbf{v_1}; \mathbf{v_2}\}$?

   (d) Is $\mathbf{0}$ in the span of $\{\mathbf{v_1}; \mathbf{v_2}\}$?

   (e) Is $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ in the span of $\{\mathbf{v_1}; \mathbf{v_2}\}$?

   (f) Write down five different vectors in the span of $\{\mathbf{v_1}; \mathbf{v_2}\}$.

   (g) What does the entire span of $\{\mathbf{v_1}; \mathbf{v_2}\}$ look like?

   (h) Does every pair of vectors in $\mathbb{R}^3$ span something similar?

   (i) Can two vectors span $\mathbb{R}^3$?

4. Below are drawn vectors $\mathbf{u}$,$\mathbf{v}$, and $\mathbf{w}$ in $\mathbb{R}^2$.

   [picture omitted]

   Is $\mathbf{w}$ in the span of $\mathbf{u}$,$\mathbf{v}$?

   Is every vector in $\mathbb{R}^2$ in the span of $\mathbf{u}$,$\mathbf{v}$?