Math 355.1 Summer 2006 Instructor: Wei Zhu **Final**

Instructions: You have **two hours** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

You must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. On the front of your blue book, pledge your exam, write your name, and include the time it took you to complete the exam. The exam is due **noon, next Tuesday, June 6**. Good luck!

Honor Pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1.(5 points each)For each of the following claim, decide whether it is true or false. No justification is necessary.
(a). The following two matrices are similar to each other:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(b). There is an orthogonal set of three nonzero vectors in $\mathbb{R}^2$.
(c). A is a square matrix, if A is singular, then zero must be an eigenvalue of A.

2. (15 points)Let the linear transformation $\vec{w} = \Gamma(\vec{v})$ from $\mathbb{R}^2$ to $\mathbb{R}^3$ be defined by

$$w_1 = v_1 + v_2, w_2 = 2v_1 - v_2, \text{ and } w_3 = 2v_2$$

where $\vec{v} = [v_1\ v_2]^T$ and $\vec{w} = [w_1\ w_2\ w_3]^T$. Find the matrix $\mathbf{A}$ that represents Γ with respect to the ordered bases $B = \{\mathbf{e}_1; \mathbf{e}_2\}$ for $\mathbb{R}^2$ and $C = \{\mathbf{e}_1; \mathbf{e}_2; \mathbf{e}_3\}$ for $\mathbb{R}^3$. If we use different bases for B' and C' respectively,

$$B' = \{[1\ 0]^T, [1\ 1]^T\}$$

$$C = \{[1\ 0\ 0]^T, [1\ 1\ 0]^T, [0\ 1\ 1]^T\},$$

what is the matrix representation $\mathbf{A}'$ with respect to the new ordered bases? Check your answer by computing $\Gamma([1\ 1]^T)$ in two ways.

3.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -5 & -6 \\ -2 & 3 & 4 \end{pmatrix}$$

(a) Find the eigenvalues of A. Compute the algebraic multiplicities of them.(5 points)
(b) For each eigenvalue, find its geometric multiplicity.(10 points)

4. (10 points)

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

Find a nonsingular matrix P such that $P^{-1}AP$ is a diagonal matrix.

5. (20 points)For the matrix in number 3, i.e.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -5 & -6 \\ -2 & 3 & 4 \end{pmatrix}$$

Use the algorithm from the handout to find a Jordan matrix J and a transition matrix Q such that $J = Q^{-1}AQ$.

6. (5 points each)(a) A is a 3×3 matrix with eigenvalue $\lambda = 1$. The algebraic multiplicity of 1 is three while its geometric multiplicity is two. Without using of the algorithm, write down a Jordan matrix J which is similar to A.
(b) A is a 4×4 matrix with eigenvalue $\lambda = 1$. Its algebraic multiplicity and geometric multiplicity are four and two respectively. Without using of the algorithm, can you tell what the Jordan form of A should be? If not, can you list the possibilities of them?

7. (15 points)Apply the traditional Gram-Schmidt process to the following three vectors

$$\vec{v_1} = [1\ 2\ 0\ 2]^T, \vec{v_2} = [0\ 1\ 1\ 0]^T, \vec{v_3} = [0\ 0\ 1\ 0]^T$$

to get an orthogonal set of nonzero vectors.