Instructions: You have two hours to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

You must show your work to receive full credit. Be sure to indicate your final answer clearly for each question. On the front of your blue book, pledge your exam, write your name, and include the time it took you to complete the exam. The exam is due next Monday, in class. Good luck!

Honor Pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1. (5 points each) For each of the following claim, decide whether it is true or false. No justification is necessary.
   (1) Let $A, B, C$ be nonsingular $p \times p$ matrices, then
   $\begin{align*}
   (ABC)^{-1} &= A^{-1}B^{-1}C^{-1}.
   \end{align*}$
   (2) Let $A$ be a $p \times p$ matrix, if the homogeneous system $Ax = 0$ has infinitely many solutions, then for any $p \times 1$ column matrix $b$, the system $Ax = b$ also has infinitely many solutions.
   (3) Let $A, B$ be two $p \times p$ matrices, if $AB$ is nonsingular, then both $A$ and $B$ must be nonsingular.

2. (15 points) Use Gauss elimination to solve the following system of equations:
   \begin{align*}
   2x_1 + 4x_2 - 4x_3 &= 0 \\
   4x_1 + 3x_2 - 3x_3 &= -1
   \end{align*}

3. (15 points) For the following matrix
   \[ A = \begin{pmatrix}
   0 & 2 & 4 \\
   2 & 6 & 8 \\
   1 & 2 & 4
   \end{pmatrix} \]
   (1) Use elementary row operations to convert it into row echelon form.
   (2) What is rank($A$)?
   (3) Is $A$ singular or nonsingular?

4. (20 points) For the following matrix
   \[ A = \begin{pmatrix}
   0 & 0 & -3 & -5 \\
   1 & -2 & -6 & -7 \\
   -1 & 3 & 7 & 7 \\
   0 & -1 & -3 & -3
   \end{pmatrix} \]
Use the method of multiple-augmented matrix to find the inverse of $A$. Then solve the following system of equations

$$Ax = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 3 \end{pmatrix}$$

by using the inverse of $A$. Do not use Gauss elimination to solve this system.

5. (20 points) (1) Find the general $3 \times 1$ solution $h$ to $Ah = 0$, where

$$A = \begin{pmatrix} -2 & -3 & 0 \\ -3 & -4 & 1 \\ 1 & 2 & 1 \end{pmatrix}.$$ 

(2) Given that $x_0 = [-1 \ 2 \ 1]^T$ solves $Ax = b = [-4 \ -4 \ 4]^T$, find the general solution to $Ax = b$ for this $b$.

6. (15 points) Find the determinant of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix}$$

(Hint: Use the row operations)