Math 355.1 Summer 2006 Instructor: Wei Zhu **Exam #2**

Instructions: You have **two hours** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

You must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. On the front of your blue book, pledge your exam, write your name, and include the time it took you to complete the exam. The exam is due **next Tuesday, in class**. Good luck!

Honor Pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1.(5 points each)For each of the following claim, decide whether it is true or false. No justification is necessary.
(1) The following four vectors are linearly independent in $\mathbb{R}^3$:

$$\begin{pmatrix} 1 \\ 3.4 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3.4 \\ 5.6 \end{pmatrix}, \begin{pmatrix} 3.14 \\ 4.4 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ -10 \\ -9.8 \end{pmatrix}.$$

(2) There is a linear transformation from $\mathbb{R}^2$ onto $\mathbb{R}^3$.
(3) For a $k-$dimensional vector space $\mathcal{V}$, if $\{\vec{v_1}, \vec{v_2}, \cdots, \vec{v_k}\}$ is linearly independent, then it is a basis of this vector space $\mathcal{V}$.

2.(15 points)The sets

$$B = \{1; t; t^2\} \text{ and } B' = \{1+t; 1-t; 1+t^2\}$$

are both ordered bases for the space $\mathcal{P}^3$ (which contains all polynomials of degree strictly less than 3, having real coefficients). Let $\vec{v} = 7 - 3t + 4t^2$.
(a) What is $C_B(\vec{v})$?
(b) Find the matrix M that translates between coordinates with respect to these two bases.
(c) Use the matrix M to find $C_{B'}(\vec{v})$.

3. (15 points) Find a basis of the subspace V_0 of $\mathbb{R}^3$ spanned by

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

4. (15 points) Extend

$$S = \{ \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \}$$

to form a basis for the subspace V_0 of $\mathbb{R}^4$ consisting of all those vectors

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

satisfying $x_1 + x_2 = x_3 + x_4$.

5. (10 points) Determine whether the two sets S_1 and S_2 below span the same subspace of $\mathbb{R}^3$.

$$S_1 = \{ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \}, S_2 = \{ \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} \}$$

6. Let $\mathcal{V}$ equal $\mathcal{P}^2$, the space of polynomials of degree strictly less than 2, having real coefficients, and let $\mathcal{W}$ equal the analogous space $\mathcal{P}^3$. Define Γ from $\mathcal{V}$ to $\mathcal{W}$ so that for each polynomial f in $\mathcal{V}$, $\Gamma(f)$ is the polynomial in $\mathcal{W}$ whose value at t equals $f(t) \cdot t$.

(a) (5 points)Verify that Γ is a linear transformation from $\mathcal{V}$ to $\mathcal{W}$.

(b) (5 points)What is the null space of Γ? What is the dimension of the image space of Γ?

(c) (5 points) Find the matrix representation of Γ with respect to the ordered bases

$$B = \{1; t\} \text{ for } \mathcal{V},$$

and

$$C = \{1; t; t^2\} \text{ for } \mathcal{W}.$$

(d) (15 points) Suppose we take instead the new ordered bases

$$B' = \{1 + t; 1 - t\} \text{ for } \mathcal{V},$$

and

$$C' = \{1; 1 + t; 1 + t^2\} \text{ for } \mathcal{W}.$$

What is the new matrix representation under the new bases? I want you to use the theorem about change of basis. (Namely, try to find out those two matrices S and P)