Topology Qual Syllabus

Point-set topology is assumed as undergraduate mathematics. The assumed material is: topological spaces, open and closed sets, basis for a topology, special topologies such as the product topology, metric topologies and the quotient topology, continuous maps, compactness, connectedness, the countability and separation properties. Specifically see: James Munkres, "Topology" (2nd edition), Chapter 2 and Sections 23, 26, 30, 31, and 32.

Fundamental group, homotopy, homotopy equivalence, statement of classification theorem of compact surfaces, Seifert-Van Kampen theorem, free groups, free products of groups, presentation of groups, covering spaces (relations with π_1, classification, deck translations, lifting criteria, relations with group actions), definition of higher homotopy groups and behavior under covering maps, statement of the theorem that a map between CW complexes that induces isomorphisms on all homotopy groups is a homotopy equivalence.

Smooth manifold, smooth map, tangent vectors and tangent space, derivative, statement of and applications of inverse function theorem and Sard's theorem, regular values, transversality, orientation, degree of smooth maps, index, vector fields.

Simplicial complex, CW-complex, singular, simplicial and cellular homology and cohomology theories, effect on homology of adding a cell, universal coefficient theorems, cup products, Poincare-Lefshetz duality, relationship between π_1 and H_1, Euler characteristic, cohomology rings of common manifolds, Kunneth theorem, statements of the Hurewicz theorem and Whitehead theorem (see Lecture Notes in Algebraic Topology by Davis and Kirk, Theorem 6.66 part 1, and Corollary 6.69 part 3 and Corollary 6.70).

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