Syllabus for Topology Qualifying Exam

Outline of Major Topics: Well-ordered sets, Zorn's lemma, topological spaces, continuous maps, compactness, connectedness, separation properties, countability properties, special topologies on metric spaces and function spaces. Fundamental group, homotopy equivalence, Seifert-Van Kampen theorem. Classification of surfaces; covering spaces (classification of, relations to π_1, group of deck translations, mondromy, relations to quotients by group actions); free groups, free and direct products of groups, presentation of groups by generators and relations; definition of higher homotopy groups and very basic properties; Hurewicz theorem and Whiteheads theorem (see study guide); manifold and orientation of manifold; simplicial complexes, CW-complex, singular homology, Mayer-Vietoris, cellular homology, singular cohomology, cup-products, Poincaré-Lefshetz duality, relationship of π_1 to H_1, Euler characteristic, cohomology rings of common manifolds, universal coefficient theorems for homology and cohomology.

Please see "Study Guide for Topology Qualifying Exam" for suggested sources.

Revised 5/27/93
Tim D. Cochran

STUDY GUIDE FOR TOPOLOGY QUALIFYING EXAM

I. Topology: A first course, James R. Munkres, Prentice Hall, 1975
Required: Chapter 1 except replace 1-11 by Zorn's Lemma

Chapter 2, Chapter 3, Chapter 4, 5.1 statement of theorem, 7.1, 7.3, 7.4, 7.5.

Chapters 8.1 through 8.8, 8.11 and 8.14 are also required but are subsumed by Massey. Exercise 10 page 342 and Exercise 4 p. 398 are crucial as is Lemma 14.2. Many times Munkres has a better treatment than Massey but Munkres is incomplete. In general the exercises in Munkres are very educational.

II. Algebraic Topology: An Introduction, William S. Massey, Springer-Verlag, 1989.

Required: 1.1 - 1.12, Chapter 2, Chapter 3 (see also: Combinatorial Group Theory, W. Magnus and A. Karrass and Solitar, Dover Publs.) 4.1 - 4.5, 5.1 - 5.11 (pay attention to 5.8).
Strongly Recommended: 6.1 - 6.7 for finite graphs, 7.1 - 7.4.

III. Elements of Algebraic Topology, James Munkrees, Addison-Wesley 1984.
Required: Fundamental Structure Theorem for finitely-generated abelian groups, 1.1 - 1.7, 1.9, 29, pp. 71 - 72, pp. 130 - 132, relative singular homology p. 168, 26, 24, 30 including motivation from pages 64 - 66 and 72 - 74, 31 and whatever needed from 15; 33, effect on homology of adjoining a cell, 38, Euler characteristic, 39, 40, skim 28, 41, pp. 262 - 267 ignoring simplicial, pp. 276 - 278, 47, 49, 50, pp. 307 - 310, 52, Thm. 53.1, 54, 55, statement of Thm. 59.3, 59.4, manifolds, orientation, 65, whatever is necessary to understand and use theorems 65.3, 67.2, 68.1 and 70.7; cohomology rings of $\mathbb{C}P(\infty)$ and $\mathbb{R}P(\infty)$.
Recommended: 73, page 125.

IV. Algebraic Topology: A first Course, M. Greenberg and J. Harper, Addison-Wesley 1981.
Required: 12, 20, 21.

V. Homotopy Theory: Sze-Tsen Hu, Academic Press 1959.
Required: definition of higher homotopy groups pp. 107 - 109, property V page 117, property VII p. 119, covering map induces isomorphism on higher homotopy groups, pp. 146 - 148 (Turewicz Thm.)
Recommended: p. 125 - 126 (role of base point.)

VI. Elements of Homotopy Theory: George W. Whitehead, Springer-Verlag 1978. Whitehead's Theorem 7.15, p. 182 (see also 7.13).

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