RICE UNIVERSITY TOPOLOGY QUAL AUGUST, 2009

Time limit 3 hrs. Closed book and notes. Concentrate on good exposition with clear and complete answers. At the end of the exam, write and sign the Rice Honor Pledge: I have neither given nor received unauthorized aid on this exam.

1. Let S_g be the closed, orientable surface of genus g.
(a) Show that for $g \geq 2$, S_g is a covering space of S_2.
(b) Is the degree of the covering (in (a)) determined by g? If so what is this degree? If not give examples.

2. Show that the only non-trivial group which can act freely and properly on an even dimensional sphere , $\mathbb{S}^{2n}$, is the group of order two.

3. Let C_n be the space obtained by attaching a 2-cell $\mathbb{B}^2$ to $\mathbb{S}^1$ by the map

$$f : \partial\mathbb{B}^2 \to \mathbb{S}^1, \text{given by } f(z) = z^n$$

(a) Describe $\pi_1(C_n)$ and $H_*(C_n)$.
(b) Do the same for $X = C_n \cup_{\mathbb{S}^1} C_m$ – the space obtained by identifying the copies of $\mathbb{S}^1$ in each. (n, m arbitrary positive integers.)

4. Determine the integral homology and cohomology groups of $X = \mathbb{CP}(2) \times \mathbb{S}^1$.

5. Describe a 1-dimensional complex on which the symmetric group Σ_3 acts freely and properly. Also describe the action and the quotient space.

6. Prove that every continuous map $f : \mathbb{S}^3 \to \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$ has degree zero.

7. Suppose M is a compact, orientable 3-dimensional manifold such that $H_i(W; \mathbb{Q}) \cong H_i(\mathbb{B}^3; \mathbb{Q})$ for all i. Prove that ∂W is a non-empty disjoint union of 2-spheres.