RICE UNIVERSITY TOPOLOGY QUALIFYING EXAM - AUGUST 2012

This is a 3-hour closed book, closed notes exam. Please show all of your work. Sign the Rice Honor pledge upon completion: I have neither given nor received unauthorized aid on this exam.

1. a) Briefly describe all possible connected covering spaces of $S^1 \times \mathbb{RP}(3)$. Include whether or not they are regular, and include their groups of covering translations.

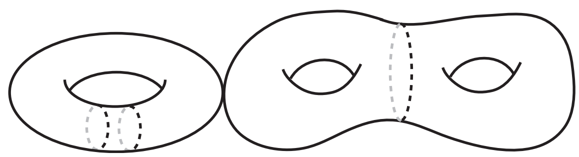
   b) Calculate $\pi_2(S^1 \times \mathbb{RP}(3))$.

2. Let S_g be the closed, orientable surface of genus g.

   (a) Show that for $g \geq 2$, S_g is a covering space of S_2.

   (b) Is the degree of the covering (in (a)) determined by g? If so what is this degree? If not give examples.

3. Suppose Y is the space obtained from the wedge of a torus and a genus two surface (pictured below),by adjoining three 2-dimensional disks, two along meridional circles of the torus, and the third along the "waist" circle of the genus two surface. The three attaching circles of these disks are the dashed circles. The three disks are not pictured.



   a) Show that Y has the structure of a CW complex and derive the corresponding cellular chain complex.

   b) From this chain complex, compute $H^p(Y;\mathbb{Z})$ for all p.

   c) Calculate $\pi_1(Y)$.

4. (a) Suppose Y is a compact, orientable n-dimensional manifold (possibly with boundary). Prove that $H_{n-1}(Y;\mathbb{Z})$ is torsion-free.

   (b) Let $f : H \to S^3$ be a topological embedding where H is a solid handlebody of genus 2 (i.e. a thickened wedge of circles) as in Figure 1. Let $\text{int}(f(H))$ be the interior of the image of f in S^3. Compute $H_p(S^3 - \text{int}(f(H)))$ for all p. Hint: $S^3 = (S^3 - \text{int}(f(H))) \cup f(H)$.

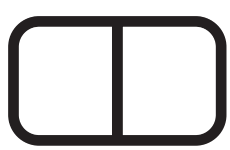


Figure 1. H, a solid handlebody of genus 2

5. Sketch the proof of the existence of the long exact sequence in homology for the pair (X, A).

6. If n is even, prove that there is no orientation-reversing (that is, degree -1) homotopy equivalence $f : \mathbb{C}P(n) \to \mathbb{C}P(n)$.